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A praxeological analysis of functions in lower secondary school: Comparing the textbooks in Japan and Indonesia

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ABSTRACT

This study undertakes a comparative analysis of the learning of functions in Japanese and Indonesian curricula. Using praxeology, a primary construct of the anthropological theory of the didactic, this study analyzed how functions are approached in both countries' school mathematics textbooks. The analysis results revealed a noteworthy contrast: while the Japanese textbooks predominantly define functions as relationships between changing quantities, Indonesians are heavily influenced by mapping elements between two sets. These findings were further explained by how the two countries' knowledge of functions is adapted. In Japan, the notion of functions evolved from proportions, commencing with the modelling of proportional relationships. In contrast, Indonesian textbooks derive functions from relations, with both concepts are introduced after set theory. This study extended its discussion upon the implication of these findings, suggesting an alternative praxeological model for inquiry-based learning about functions.

Keywords: praxeology, the anthropological theory of the didactic, functions, textbooks, comparative study

INTRODUCTION

The mathematical knowledge taught at schools, according to Chevallard (2019), undergoes a distinct transformation from its original form so that it can be seamlessly integrated into the school system (Bosch & Gascón, 2006). Starting from this idea, Chevallard introduced the didactic transposition: the process encompasses the transformation of knowledge from scholars to knowledge to be taught, taught knowledge, until learned knowledge (Bosch & Gascón, 2014; Chevallard & Bosch, 2020; Lundberg & Kilhamn, 2018). The manifestation of knowledge to be taught is primarily represented in the school curriculum, often reflected in school textbooks. The textbooks are commonly called 'the potentially implemented curriculum' because they serve as a crucial media for teachers and students (Pepin et al., 2013; Sosniak & Perlman, 1990; Valverde et al., 2002). Despite the actual learning being reflected in 'taught knowledge', teachers worldwide rely on textbooks as their primary teaching sources (Mullis et al., 2012). Given their importance, textbook analysis has been widely acknowledged as a research field in mathematics education (Fan et al., 2013).

Numerous studies have focused on examining mathematics textbooks, spanning national and international contexts. On a national level, González-Martin et al. (2013) investigated the introduction of real numbers in Brazilian textbooks, revealing that the mathematical needs for constructing real numbers remained unclear. Internationally, comparative analyses of textbooks have been conducted. Takeuchi and Shinno (2020) reported that geometric proofs influenced the learning of symmetry and transformations in Japanese textbooks, while in England, the topics had many connections to other mathematical domains. Additionally, Hidayah and Forgasz (2020) identified variations in geometry topics across Indonesian and Australian textbooks, which suggested that both countries' textbooks should provide more non-routine and open-ended tasks. While geometry contents analysis became popular among previous comparative studies, this current study aims to contribute to the existing literature by conducting a comparative analysis of mathematics textbooks, seeking insights into how specific mathematical topics in algebra are organized across different countries.

For conducting a comparative study, the anthropological theory of the didactic (ATD) by Chevallard (2006, 2019) serves as the theoretical framework for the textbook analysis. In specific, this study focuses on the transposition process from scholarly knowledge to knowledge to be taught, which is known as external didactic transposition (Bosch & Winsløw, 2020). Within the ATD, knowledge in various countries is presented by examining the conditions and constraints within each educational institution. The production of textbooks is viewed as one of the results of didactic transposition. In this study, praxeology, a fundamental construct within ATD, is employed as an analytical tool to investigate how specific mathematical knowledge is situated in different countries. Praxeology enables the examination of tasks and techniques within textbooks intended to build knowledge about a specific mathematical topic, shedding light on the theoretical discourse (or the mathematical knowledge such as concepts, theorems, and others) underlying the chosen technique.

This study investigated how functions are addressed in the curricula of two specific countries with different educational systems: Japan and Indonesia. The choice of comparing both countries' curricula stems from two considerations. Firstly, the concept of functions had undergone a significant shift in pre- and post-modern mathematics reforms. Functions in pre-modern mathematics were based on the dependency of one quantity to another. Thus, the concept focused on the relationship between changing variable quantities. In post-modern mathematics, with the introduction of set theory, the concept of function was not again restricted to variable relationships but shifted to pairings of elements between two sets and thus became the formal definition of functions until recent days. Secondly, the recent mathematics curriculum in Japan has initiated a structured problem-solving approach to mathematics learning. Hence, the curriculum reinstates the original concept of functions as the relationship between changing quantities since the concept is closely related to problem-solving in real-life contexts. Meanwhile, the mathematics curriculum in Indonesia focuses on ensuring students understand formal mathematics concepts, such as what a function is, before applying the concept to practical problems. For this reason, functions in Indonesia are introduced as mapping elements between two sets since the formal concept of function is derived from set theory. Considering this, valuable insights can be gained by comparing the textbooks of these countries.

Furthermore, the use of praxeology in textbook research on this topic remains relatively new. While Wijayanti (2018) employed praxeology to examine linear functions in Indonesian textbooks, our study takes a broader perspective, scrutinizing how functions are defined, represented, and interconnected with various mathematical topics within the Japanese and Indonesian curricula. Through a praxeological analysis, this study explored the different presentations of the function topic in textbooks from Japan and Indonesia. Following the aim of this study, the research question addressed is, as follows: what are the similarities and differences between learning functions in Japanese and Indonesian mathematics textbooks?

THEORETICAL PERSPECTIVE

Functions and Function Representations

Exploring the historical perspective of functions is a foundational aspect of this study, given the varied manifestations of functions across different representations. The development of the notion of function witnessed two primary conceptions. Before the modern mathematics era, functions were perceived as relationships between varying quantities, with one quantity dependent on another (Biehler, 2005). This relationship found expression through numerical tables, graphs, and formulas (Kleiner, 1989, 1993; Ponte, 1992). This conception was recognized as the old "algebraic" conception due to its concrete, analytic, and constructive application of a function. However, the former conception was considered too practical and could not capture certain relationships between variables, whether in numbers or objects, in which the relationship cannot be expressed algebraically. Based on this reason and coincided with the introduction of set theory, the conception of functions evolved to incorporate the set definition: a correspondence of elements between two non-empty sets, where each element in the first set was assigned to an element in the second set (Vinner & Dreyfus, 1989). This new "logical" conception of a function emerged in post-modern mathematics and became a function's abstract or formal definition.

In learning functions, Sfard (1991) emphasized the dual nature of function conceptions, encompassing the operational view—function as a computational process transforming one element into another—and the structural view—function as a mathematical entity within its domain. These conceptions are reflected in different representations, with tables highlighting the process, graphs as objects, and formulas establishing connections between both (Ronda, 2015; Tall, 1993; Sfard, 1991). Recognizing the significance of understanding both conceptions, Doorman et al. (2012) introduced the three aspects of a function. The aspects spanning from a function as concrete to an abstract object encompass 1) a function as an input-output assignment, 2) a function as a dynamic covariation process, and 3) a function as a mathematical object.

As this study will see in more detail, in lower secondary school, the notion of function is introduced through the linear functions, which are functions denoted by $f(x) = ax + b$, where a and b are fixed numbers (Wijayanti, 2018). Moreover, to identify how function representations unfold in the textbook, this study adopted techniques commonly employed in function learning, as Mesa (2004) outlined. The technique includes actions related to the known relation–do something with the relation, such as locating points in a graph and finding an element of the range, and actions about an unknown relation–do something about the relation—like finding the relation and describing graph shapes. Furthermore, the techniques employed can also be categorized into quantitative and qualitative analyses (Rolfes et al., 2018). Quantitative analysis involves numerical manipulations of functions, while qualitative analysis focuses on functions as an object.

Ultimately, an essential aspect to investigate is whether the textbook effectively fosters the understanding of connections between different function representations. Bardini and Stacey (2006) noted that the parameters a and b in $f(x) = ax + b$ hold distinct meanings within each representation. In numerical contexts, such as tables, a signifies the rate of change, while b is the value of $f(x)$ when $x = 0$. In symbolic representations, like algebraic expressions, a denotes the coefficient of x, and b is the constant term. Lastly, in graphical representations, a corresponds to the gradient of the graph, and b signifies the y-intercept.

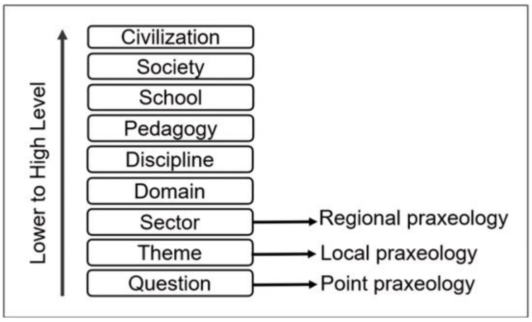

Figure 1. The levels of didactic co-determinacy (Source: Authors' own elaboration)

Praxeology

In analyzing the textbooks of Japan and Indonesia, this study adopted the concept of praxeology, a central theoretical framework within the ATD. According to Chevallard (2006), praxeology comprises two interrelated blocks: *praxis* and *logos* (Bosch & Gascón, 2014). The *praxis* block consists of a type of task (T), which is executed through a particular technique (τ) or a way to solve the task. *Praxis* remains incomplete without rationale or justification for why such a technique is used and not another (Chevallard et al., 2015). Consequently, the *praxis* block finds justification in the *logos* block. The block encompasses a technology (θ)–a term derived from *techno* and *logos*, denoting the knowledge of the technique (τ); and a theory (Θ)–a more abstract discourse justifying the technology (θ). The amalgamation of this quadruplet $[T/\tau/\theta/\Theta]$ constitutes a praxeology $\wp$.

Furthermore, the organization of praxeology can be differentiated based on the activities or knowledge in question: a point praxeology (a single $T/\tau/\theta/\Theta$), a local praxeology (encompassing a set of T/τ sharing a common θ and a Θ), and a regional praxeology (consisting of all point and local praxeologies justified by a common Θ).

It is also essential to consider that each institution, represented by Japanese and Indonesian schools in this study case, possesses a peculiar notion of mathematical knowledge. The notion subjects to transposition that shapes the knowledge within the conditions and constraints of the institution's existence (Chevallard et al., 2015). Considering this aspect, the ATD introduces another construct, the level of didactic co-determinacy (Artigue & Winsløw, 2010; Bosch & Gascón, 2014), which proves valuable in elucidating the organization and distinct limitations of knowledge within each institution, as depicted in **Figure 1**. The didactic co-determinacy can be helpful for international comparative studies (Artigue & Winsløw, 2010). From this perspective, the similarities and differences observed at a specific level can be explained by the conditions prevalent at higher levels (Takeuchi & Shinno, 2020).

MATERIALS AND METHODS

In constructing and implementing the praxeological analysis for this study, empirical data were extracted from textbooks used in Japanese and Indonesian lower secondary schools. The textbook selection serves as an illustrative model, enabling the exploration of important questions concerning the organization of practical knowledge for student engagement. This study foresees two noteworthy contributions: firstly, it offers a comprehensive insight into the introduction of function in schools, particularly in Japanese and Indonesian educational settings; secondly, it provides a replicable demonstration of praxeology's application in textbook analysis. The explicit use of praxeology for comparing textbooks is intended to be a valuable reference for other researchers seeking to conduct similar analyses grounded in this theoretical framework.

Data Collection: The Textbook Selection

The primary data analyzed were derived from lower secondary mathematics textbooks. The Japanese textbooks under examination included *Keirinkan's gateway to the future: Math 1* (Okamoto et al., 2013) for grade 7 and *Keirinkan's gateway to the future: Math 2* (Okamoto et al., 2014) for grade 8, one the most popular textbook series published in Japan. The Japanese textbooks were originally written in Japanese; however, this study used the English translation. The textbook used for Indonesia was *Matematika SMP/MTs kelas VIII* [*Mathematics for grade 8 lower secondary school*] (Tohir et al., 2022), the official textbook released by the Indonesian Ministry of Education. Although Indonesian publishers publish multiple versions of textbooks, all adhere to the same learning objectives outlined in the official Indonesian curriculum. This alignment ensures a consistent sequence of tasks (or problems) across textbooks, making this study's official textbook a representative choice.

Furthermore, different educational backgrounds are also considered in this study, particularly when selecting the textbooks used. Japan's educational system is typically revised every ten years under the national curriculum called the *Courses of Study*, with the latest revision made in 2018 for the lower secondary school level (Ministry of Education, Culture, Sports, Science and Technology [MEXT], 2018). Although the selected Japanese textbooks are based on the previous national curriculum (MEXT, 2008), the functions learning in the 2018 curriculum still need a revision from the previous one. Additionally, the English translation of the newest textbooks has yet to be distributed. The selected textbook is the latest edition for Indonesia, reflecting the most recent official curriculum (Ministry of Education, Culture, Research, and Technology [Kemdikbud], 2024). Additionally, the official Indonesian textbook is readily accessible online at https://buku.kemdikbud.go.id/. Given this information, they were appropriate as the primary sources in this study.

Table 1. Topics and learning objectives of linear functions in the Japanese and Indonesian textbooks

Japan		Indonesia	
Grade/chapter	**Objectives**	**Grade/chapter**	**Objectives**
7th grade/ change and correspondence	To learn more about the relationship between pairs of quantities that change together; to express proportional relationships using tables, expressions, and graphs; to express inversely proportional relationships using tables, expressions, and graphs; to use proportional and inversely proportional relationships to solve real-world problems		
8th grade/ linear functions	To learn more about the relationship between two quantities that change together; to express linear functions using tables, expressions, and graphs; to use the graph of linear functions to find the expression; to use linear functions to solve real-world problems	8th grade/ relations and functions	To understand the concept of set, relation, and function; to give examples of set, relation, and function in daily-life activities; to represent relations and functions in different forms; to find the function values and function graphs in cartesian coordinates; and to solve problems related to relations and functions

Table 1 presents general information about the grade, chapter, and learning objectives of linear functions in Japanese and Indonesian textbooks. The "chapter" refers to the chapter's name, where function topics are in the textbooks. At the same time, "objectives" outline the expected learning outcome following the chapters' completion, aligning with each country's curriculum objectives. As illustrated in **Table 1**, the organization of the function learning in Japanese textbooks differs from that in the Indonesian textbook. In Japan, the introduction of function occurs in grade 7, followed by the study of linear functions in grade 8. Conversely, in Indonesia, both the introduction of function and linear functions are covered within the same chapter studied in grade 8.

Data Analysis

The data analysis process unfolded in two phases. Initially, the focus was on the practical aspect of the textbooks before transitioning to the theoretical one. In the practical or *praxis* block, this study identified all questions (problems) posed in Japanese and Indonesian textbooks, later defining these questions as 'tasks' by praxeology. These tasks were found in both countries' textbooks, typically presented as 'example' and 'exercise' questions. While recognizing that some of these questions may not be completed in the classroom but rather assigned as homework or studied independently by students, all questions were included in the analysis as they reflect the curriculum of Japan and Indonesia. Subsequently, the technique used to solve each task was categorized according to the techniques outlined by Mesa (2004) for working with functions. These techniques revolve around 'doing something with the relation' or 'doing something about the relation,' with each category elaborated in detail in the results section.

To analyze the knowledge or *logos* block, this study referenced the definitions, properties, and conceptions of functions as outlined in each textbook. In addition to these analyses, the *logos* component of praxeology explored in the textbooks will be further explained by identifying the conditions and constraints of learning functions in Japan and Indonesia using didactic co-determinacy. This approach aimed to unveil the similarities and differences in how functions, as mathematical knowledge, are structured within each educational system.

RESULTS

In this section, the study considers the praxeological organization of functions within the Japanese textbooks first and then the Indonesian textbook.

The Praxeological Organization of Function in Japanese Textbooks

The initial examination of the praxeological organization began with identifying the *praxis* block of the Japanese textbooks. This study attempted to categorize a set of questions sharing similar characteristics into a type of task. This process identified 16 types of tasks from two textbooks (grade 7 and grade 8) and labelled from T_{JP1} to T_{JP17}. The task types T_{JP1} to T_{JP10} belong to the grade 7 textbook, while the rest belong to the grade 8 textbook. Along with the task types, the techniques used were also described and labelled from τ_{JP1} until τ_{JP21}. Note that each type of task can be solved using more than one technique, or different types of tasks can be solved using a similar technique. The analysis result of the praxis block of functions in the Japanese textbook grade 7 is described in **Table 2**, as follows.

According to **Table 2**, the learning of functions in Japan came from proportionality, which comprised two main topics: proportional and inversely proportional relationships. The type of task T_{JP1} to T_{JP5} emphasized working with (direct) proportional relationships, which the textbook named 'proportional relationships'. At the beginning of the learning, the textbook defined proportional relationships as relationships expressed by $y = ax$. Following this definition, tasks centered on representing proportional relationships are addressed. The form of representation started from tables (T_{JP2}), which can be solve by listing the

Table 2. *Praxis* block of functions in the Japanese textbook grade 7

Type of task	Technique
T_{JP1}: To define a proportional relationship Examples of task: "For each of the items below, check to see if y is proportional to x. (1) The total cost y yen when buying x 50-yen stamps …"	τ_{JP1}: Using $y = ax$ to identify proportional relationship in the context given
T_{JP2}: To represent proportional relationships with tables Examples of task: "Complete the table if x minutes is the amount of time since the incense was lit and y mm is the length of the portion that has burned away."	τ_{JP2}: Filling the table (with quantities denoted by x as the independent variable and y as the dependent variable)
T_{JP3}: To identify the changes of variables in proportional relationships Example of task: "For each of the items below, check to see if y is proportional to x. Then, state the constant of proportion in each. (1) the total cost y yen when buying x 50-yen stamps …"	τ_{JP3}: Using the formula $\frac{y}{x} = a$
T_{JP4}: To represent proportional relationships with formulas Example of task: "Look at the incense-burning experiment. Express the relationship between x and y …" and "y is proportional to x, and $y = 16$ when $x = 8$. Write an expression indicating the relationship between x and y."	τ_{JP4}: Using the values listed on the table to find the formula denoted by $y = ax$, τ_{JP5}: using substitution method of equation $y = ax$
T_{JP5}: To represent proportional relationships with graphs Example of task: "Plot the graph $y = 1.5x$ on the figure above …" and "Plot the graph $y = -1.5x$ on the figure above …"	τ_{JP2}, τ_{JP6}: Locating points (x_n, y_n) on the cartesian plane, τ_{JP7}: Increasing the number of points (x_n, y_n) on the plane to make a line
T_{JP6}: To define an inversely proportional relationship Examples of task: "In which of the following is y inversely proportional to x? (1) The base x cm and the height y cm of a triangle with an area of 6 cm^2 …"	τ_{JP8}: Using $y = \frac{a}{x}$ to identify proportional relationship in the context given
T_{JP7}: To represent inversely proportional relationships with tables Examples of task: "Complete the table if x is the width and y is the height."	τ_{JP2}
T_{JP8}: To identify the changes of variables in inversely proportional relationships Examples of task: "… As the value of x doubles and triples, what happens to the value of y?"	τ_{JP9}: Identifying that the relationship between x and y can be expressed as $xy = a$
T_{JP9}: To represent inversely proportional relationships with formulas Examples of task: "Draw various rectangles with an area of 6 cm^2. Express the relationship between x and y …" and "y is inversely proportional to x, and $y = 2$ when $x = 4$. Write an expression indicating the relationship between x and y."	τ_{JP10}: Using the values listed on the table to find the formula denoted by $y = \frac{a}{x}$, τ_{JP11}: using substitution method of equation $y = \frac{a}{x}$
T_{JP10}: To represent inversely proportional relationships with graphs Example of task: "Draw the graph $y = \frac{6}{x}$" and "Draw the graph $y = -\frac{6}{x}$."	τ_{JP2}, τ_{JP6}, τ_{JP7}

Note. T_{JP}: Type of task in Japanese textbooks & τ_{JP}: Technique used in Japanese textbooks

value of x and y in **Table 2** (τ_{JP2}). The subsequent task focused on identifying changes in proportional relationships (T_{J3}) by dividing the value in y with the value in x (τ_{JP3}). Although task pertaining to generating formulas (T_{JP4}) already introduced in the beginning of the learning, i.e., by conducting generalisation of the relationship listed in **Table 2** (τ_{JP4}), there were other tasks of this type which can be solve by doing substitution to $y = ax$ (τ_{JP5}). The last representation is graph (T_{JP5}), learned gradually from locating points on the plane (τ_{JP6}) to increasing the number of points so that the graph become a straight line (τ_{JP7}). The next topic, about inversely proportional relationships ($T_{JP6} - T_{JP10}$), was presented similarly to the task types of the direct ones. In the Japanese textbook, an inversely proportional relationship was denoted by $y = \frac{a}{x}$ or $xy = a$.

Furthermore, function learning in Japan continued in grade 8. While functions in grade 7 is called proportional relationships, functions learned in grade 8 are called linear functions. Note that the textbooks in Japan distinguish between proportion functions $y = ax$ with linear functions $y = ax + b$ though both are 'linear functions' or 'functions with first degree'. **Table 3** shows the praxis block of functions presented in the grade-8 textbook of Japan.

Based on **Table 3**, the task sequence of linear functions is also similar to tasks about proportional relationships. The textbook began with identifying examples and non-examples of linear functions (T_{JP11}). It continued to tasks about representing linear functions with tables (T_{JP12}), identifying the rate of change (T_{JP13}), and then representing linear functions with formulas $y = ax + b$ (T_{JP14}). Moreover, a notable difference was identified in linear function graph tasks (T_{JP15}). The function graph tasks in grade 8 textbook were addressed more detail and the techniques used to solve T_{JP15} were also varied. Graphing a linear function did not begin by making tables and then plotting points on the cartesian plane; instead, it used the knowledge of graphing $y = ax$ to obtain the graph of $y = ax + b$, resulting in a more advanced technique (τ_{JP16}). There are also techniques to build connection between the line slope with the rate of change (τ_{JP17}) and the y-intercept with the constant b (τ_{JP18}). The task type T_{JP16}, which appeared in the grade 8 textbook, entails the connection between linear function formulas and graphs. The tasks included in this type asked about constructing the linear function formula from the graph, whether solved by substitution of the points (x, y) and the slope a (τ_{JP19}) in $y = ax + b$ or substitution of the two points (x_1, y_1) and (x_2, y_2) in $y = ax + b$ and elimination of two simultaneous equations (τ_{JP20}). The last task type-technique summarized the relationship between function tables, formulas, and graphs (T_{JP17}/τ_{JP21}).

Furthermore, to interpret the *logos* block in Japanese textbooks, this study organized the set of task types and techniques around a common technological discourse. Based on this categorization, three local praxeologies were identified. The first local praxeology, $\wp_{JP1}$, comprised of the *praxis* component $(T_{JP1} - T_{JP5})/(\tau_{JP1} - \tau_{JP7})$, which shared a similar technology related to (direct) proportional function $y = ax$ (θ_{JP1}). According to the textbook, this is the initial type of function encountered by students. Since proportionality consisted of both direct and inverse proportions, which co-exist with each other, the Japanese textbook extended the learning of (inverse) proportional function $y = {}^{a}/_{x}$ (θ_{JP2}) that justified the set of $(T_{JP6} - T_{JP10})/(\tau_{JP2}, \tau_{JP6} - \tau_{JP11})$, made it into the second local praxeology ($\wp_{JP2}$). While the initial two local praxeologies were organized within the 7-grade

Table 3. Table on top of a page

Type of task	Technique
T_{JP11}: To define a linear function Examples of task: "The expressions below indicate that y is a function of x. Which are linear functions? (1) $y = -8x + 3$ …" and "In which of the following is y a linear function of x? (1) the remaining quantity of flour (y g) after using x g out of 300 g total…"	τ_{JP12}: Using $y = ax + b$ to identify linear functions in the context given
T_{JP12}: To represent linear functions with tables Examples of task: "Complete the table if x minutes represent the quantity of time that the tank has been filling and y cm the height of the water." and "Complete the table for the linear function $y = 2x + 3$ …"	τ_{JP2}
T_{JP13}: To identify the changing values in linear functions Example of task: "Complete the table to find the rate of change for the linear function $y = -2x + 7$" and "For the linear function $y = \frac{2}{3}x + 5$, find the increase in y under the following conditions. (1) When the increase in x is 1 …"	τ_{JP2}, τ_{JP13}: Using the formula for additive rate of change, namely $a = \frac{increase\ in\ y}{increase\ in\ x}$
T_{JP14}: To represent linear functions with formulas Example of task: "A tank is filled with 8 cm of water. The students start filling it at a rate of 2 cm per minute. What can you say about the relationship of x and y in this case?" and "Find the expression for the linear function if the rate of change is -5, and $y = 3$ when $x = 2$."	τ_{JP14}: Using the values listed on the table to find the formula denoted by $y = ax + b$, τ_{JP15}: using substitution method of equation $y = ax + b$
T_{JP15}: To represent linear functions with graphs Example of task: "The figure on the right shows the graphs of $y = 2x$ and $y = -2x$. Use this information to graph the linear functions (1) $y = 2x - 2$ …," "State the slope and intercept of the lines below. (1) $y = 3x - 5$ …," and "Graph the linear functions below by identifying the slope and intercept first. (1) $y = x - 3$ …"	τ_{JP16}: Drawing the graph of $y = ax + b$ from the graph of $y = ax$, τ_{JP17}: identifying that the line slope is the same as the rate of change (a) in $y = ax + b$, τ_{JP18}: identifying that the y-intercept is the same as the b in $y = ax + b$
T_{JP16}: To determine linear function formula from the graph Examples of task: "y is a linear function of x. The graph of the function is a line that passes through point (1,2) and has a slope of -3. Find the expression for linear function." and "y is a linear function of x. The graph of the function is a line that passes through point $(-1, -4)$ and point (3,8). Find the expression for linear function."	τ_{JP19}: Substituting the points (x, y) and the slope a to $y = ax + b$ to find the value of b, τ_{JP20}: substituting the points (x_1, y_1) and (x_2, y_2) to $y = ax + b$, eliminate the equations to find the value of a and b
T_{JP17}: To identify the relation between function representations Examples of task: "The above information summarizes the relationship between the table, expression, and graph for the linear function $y = 2x + 1$. If we know any one of these three, we can find out various other piece of information. For example …"	τ_{JP21}: Interpreting the a and b in $y = ax + b$ from table, formula, and graph

Note. T_{JP}: Type of task in Japanese textbooks & τ_{JP}: Technique used in Japanese textbooks

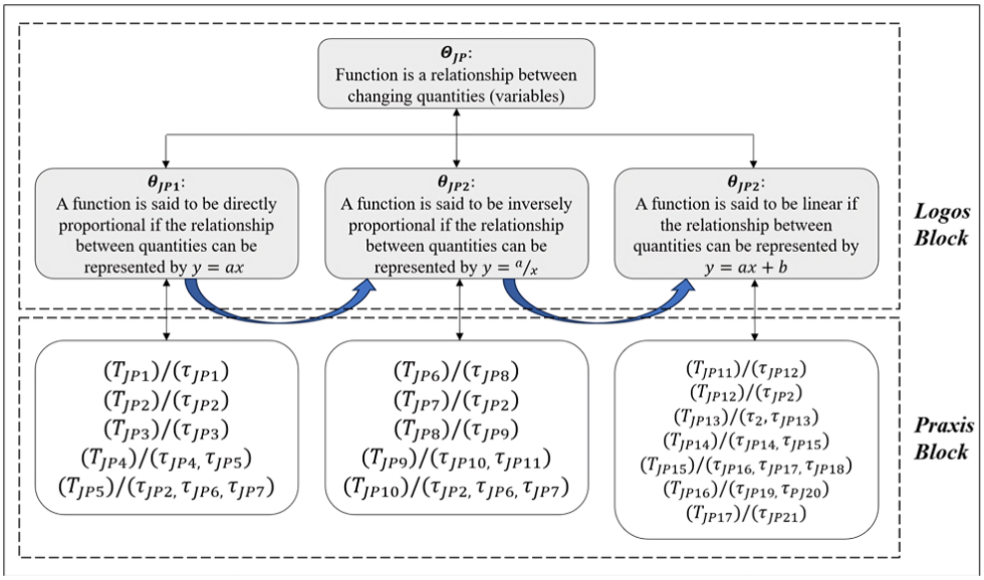

Figure 2. The praxeological model of functions in the Japanese curriculum (Source: Authors' own elaboration)

textbook, the last one, $\wp_{JP3}$, existed within the 8-grade textbook. Here the technological discourse about linear function $y = ax + b$ (θ_{JP3}) justified the set of $(T_{JP11} - T_{JP17})/(\tau_{JP2}, \tau_{JP12} - \tau_{JP21})$. All the technologies found a similar justification in theory, Θ_{JP}, about function as a relationship between changing quantities (variables).

To conclude, these local praxeologies become the praxeological (or mathematical) model of the function knowledge structured in the Japanese curriculum, as shown in **Figure 2**.

Table 4. *Praxis* block of functions in the Indonesian textbook grade 8

Type of task	Technique
T_{ID1}: To define a function Examples of task: "Let the set $P = \{3, 4, 5, 6, 7\}$ and the set $Q = \{4, 5, 7, 9\}$. 'plus one from' is the relation determined. Is the relation between P and Q a function?" and "Given the sets of ordered pairs below. (i). $\{(0, 0); (2, 1); (4, 2); (6, 3)\}$ (ii). $\{(1, 3); (2, 3); (1, 4); (2, 4)\}$ (iii). $\{(1, 5); (2, 5); (3, 5); (4, 5)\}$ (iv). $\{(5, 1); (5, 2); (4, 1); (4, 2)\}$ The above pairs which can be called as a mapping (function) is …"	τ_{ID1}: Asigning an element of a set to an element on the other set
T_{ID2}: To represent functions with sets of ordered pairs Examples of task: "Let the set $E = \{p, q\}$ and the set $F = \{2, 3, 4\}$. Represent with a set of ordered pairs a function from E to F." and "Given two sets P and Q, that is the set $P = \{0, 1, 4, 9\}$ and the set $Q = \{0, 1, 2, 3, 4\}$. Use arrow diagram to demonstrate the function from P to Q."	τ_{ID1}
T_{ID3}: To represent functions with tables Examples of task: "Given that a function f with the domain $A = \{6, 8, 10, 12\}$ and $f(x) = 3x - 4$. Represent that linear function with table."	τ_{ID2}: Filling the table (with quantities denoted by x as the element of domain and $f(x)$ as the element of range)
T_{ID4}: To represent functions with formulas Example of task: "The information below is the fare applied by a taxi company. The taxi 'flag fall' is Rp7.500 and Rp3.000 per kilometer. How can you determine the function formula for the fare?" "Given that a function $f: A \to B$ be defined with $f(x) = 4x - 3$ with $A = \{-2, -1, 0, 1, 2, 3\}$ and B is the set of real numbers. Determined the range of f." and "Given that a linear function f has value -4 when $x = -1$, and has value 5 when $x = 2$. Determine the function formula."	τ_{ID3}: Using the values listed on the table to find the formula denoted by $f(x) = ax + b$, τ_{ID4}: Using substitution method of equation $f(x) = ax + b$, τ_{ID5}: using substitution and elimination methods of simultaneous equations $f(x) = ax + b$
T_{ID5}: To represent functions with graphs Example of task: "A function f is defined with a formula $f(x) = 5 - 3x$ and the domain is $\{-2, -1, 0, 1, 2, 3\}$. Draw the graph of f."	τ_{ID6}: Finding the value of $f(x)$ from then domain given, then locating points $(x_n, f(x_n))$ on the cartesian plane
T_{ID6}: To define a bijective function Examples of task: "Each student has a unique number as listed below. Ahmad's number is 219, As'ari's number is 279, Tohir's number is 292, Wati's number is 224. If A is the set of students and the element of B is the set of student's number, then represent both relations with Venn diagrams and identify what characteristic can be seen from the relations." and "Which one of the sets of ordered pairs below is a one-to-one correspondence? Please explain. a. $\{(1, 1); (2, 2); (3, 3)\}$ d. $\{(1, p); (2, q); (3, p)\}$ b. $\{(2, 2); (2, 4); (2, 6)\}$ e. $\{(5, 6); (6, 7); (7, 5)\}$ c. $\{(a, 2); (2, b); (b, a)\}$ f. $\{(a, x); (b, z); (a, y)\}$ "	τ_{ID1}
T_{ID7}: To represent bijective functions with sets of ordered pairs Examples of task: "Given the set $K = \{1, 2, 3, 4, 5, 6\}$ and the set $L = \{a, b, c, d, e, f\}$. Try, at least 3 sets of ordered pairs, a one-to-one correspondence from the set K to the set L."	τ_{ID1}
T_{ID1}: To define a function Examples of task: "Let the set $P = \{3, 4, 5, 6, 7\}$ and the set $Q = \{4, 5, 7, 9\}$. 'plus one from' is the relation determined. Is the relation between P and Q a function?" and "Given the sets of ordered pairs below. (i). $\{(0, 0); (2, 1); (4, 2); (6, 3)\}$ (ii). $\{(1, 3); (2, 3); (1, 4); (2, 4)\}$ (iii). $\{(1, 5); (2, 5); (3, 5); (4, 5)\}$ (iv). $\{(5, 1); (5, 2); (4, 1); (4, 2)\}$ The above pairs which can be called as a mapping (function) is …"	τ_{ID1}: Asigning an element of a set to an element on the other set

Note. T_{ID}: Type of task in Indonesian textbooks & τ_{ID}: Technique used in Indonesian textbooks

The Praxeological Organization of Function in The Indonesian Textbook

In accordance with the examination of praxeology conducted in Japanese textbooks, this study initially identified the praxis component, namely the type of task and technique, in the Indonesian textbook. Since this study aimed to compare the praxeological organization of functions in the textbooks of Japan and Indonesia, the categorization of questions (or tasks) in the Indonesian textbook followed the type of task in Japanese textbooks. The tasks which did not match in any Japanese type of task were labelled differently. This process identified seven types of tasks from one textbook (grade 8) and denoted from T_{ID1} to T_{ID7}. Moreover, the techniques used to solve each task type were also described from τ_{ID1} until τ_{ID6}. Similar to the technique in the Japanese textbook, each type of task can be solved by one or more techniques, or one specific technique can be used to solve different task types. The *praxis* block of functions in the Indonesian textbook is shown in the **Table 4**.

Based on **Table 4**, the task sequence of functions in the Indonesian textbook was similar to how Japanese textbooks introduced functions. The introduction began with defining a function, including recognizing examples and non-examples of functions (T_{ID1}). A notable difference was identified in the Indonesian textbook; examples of functions given are ordered pair sets. This first type of task became the "reference" of how the textbook presented functions in the subsequent task types. The type of task from T_{ID2} to T_{ID6} emphasized different forms of function representations. Since the textbook defined functions in terms of sets, the first type of task of function representation was about functions with sets of ordered pairs (T_{ID2}), which was solved simply

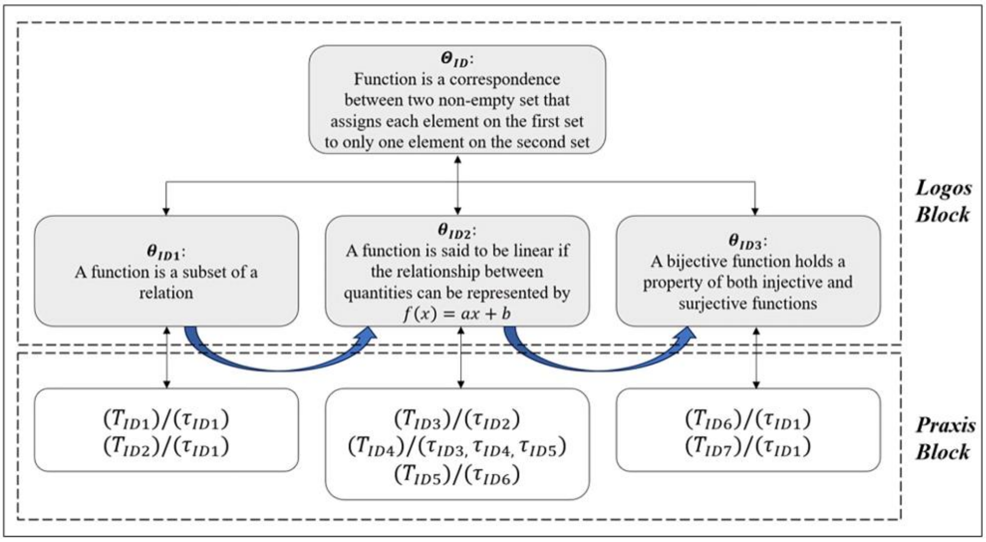

Figure 3. The praxeological model of functions in the Indonesian curriculum (Source: Authors' own elaboration)

by assigning elements of both sets (τ_{ID1}). The subsequent tasks about representations also exist in Japanese textbooks, namely function tables (τ_{ID3}), function formulas (τ_{ID4}), and function graphs (τ_{ID5}). However, unlike the Japanese textbook, which began with $y = ax$ to $y = ax + b$, the linear function presented in the Indonesian textbook learned straightly to $f(x) = ax + b$. Some techniques used were also common to the Japanese techniques; for example, τ_{ID3} and τ_{ID4}, which were parallel with τ_{JP14} and τ_{JP15}, were the techniques to solve τ_{ID4}. The last two task types (T_{ID6} and T_{ID7}) about bijective functions only existed in the Indonesian textbook, expanding functions formed by sets.

Moreover, this study also conducted the same approach to interpret the *logos* block of functions in the Indonesian textbook. The set of task type-technique components was organized into three technological discourses. However, due to the distinct approach to introducing the concept of function, the local praxeologies in Indonesia mainly exhibited different organizations from those in Japan. The first local praxeology $\wp_{ID1}$, highlighted functions as a special case of a relation (θ_{ID1}) to justify the *praxis* block $(T_{ID1}, T_{ID2})/\tau_{ID1}$. The second local praxeology $\wp_{ID2}$, comprised of the set $(T_{ID3} - T_{ID5})/(\tau_{ID2} - \tau_{ID6})$ shared a common technology about "linear functions $fx = ax + b$" (θ_{ID2}). According to the analysis, the idea of function from the first technology shifted in the second technology, i.e., from function as a mapping to function as a rule. The last local praxeology $\wp_{ID3}$, discusses bijective functions to explain the *praxis* block $(T_{ID6}, T_{ID7})/\tau_{ID1}$. In this last praxeology, function as mapping generated all examples of functions in the tasks. Considering that all functions examples tailored to the idea of functions as sets, even for domains and ranges of linear functions written as sets, all the technologies were explained by a general theory of function, Θ_{ID}, pertaining to functions as a correspondence between sets.

In conclusion, **Figure 3** depicts the praxeological (or mathematical) model of the function knowledge structured in the Indonesian curriculum.

The analysis of Japanese and Indonesian textbooks sheds light on the different praxeological organizations and how functions are situated in each country. These differences were also influenced by the conditions and constraints of the mathematics curriculum reformation in each country, which will be explained in the discussion section.

DISCUSSIONS

The following theoretical and empirical discourse of the findings will be described in alignment with the themes presented.

Regarding the concept of a function, all three local praxeologies in Japanese textbooks adhere to one general discourse, which is strongly influenced by the earlier interpretation of functions as *the dependency of a variable's quantity to another variable's quantity* and use x and y to denote the variables (Kleiner, 1989, 1993). This notion of function emerged before the modern mathematics era. In contrast, the local praxeologies in the Indonesia textbook implies functions through the set-theoretical definition. By this means, Indonesia adopted the concept developed during the modern mathematics era, where the Set Theory was first introduced, and offered a general meaning of the function itself. Both approaches offer advantages. While functions as sets encompass both numerical and non-numerical contexts (Markovits et al., 1986), functions in analytical sense are widely used for modelling real-life situations (Sierpinska, 1992).

Furthermore, learning functions are inseparable from learning the multiple representations of a function. Both Japanese and Indonesian textbooks offer tasks related to functions represented by tables, formulas, and graphs. A similarity is identified in the point praxeology in Japanese textbooks, i.e., expressing proportion and linear functions with tables and expressing functions with tables in the Indonesian textbook. Both praxeological organizations indicate functions as a calculation process from one element to another (Doorman et al., 2012), defined as the operational conception of a function by Sfard (1991)–the difference lies in the transition functions in tables to formulas. Prior to function formulas, the Japanese textbooks emphasize the idea of functions as a dynamic covariation process (Doorman et al., 2012), suggesting that changes in one quantity influence changes in another quantity. Applying the covariational approach to functions makes the concept of rate of change more visible to students (Confrey & Smith, 1994). Thus, it enables students to build a rule or formula for the function. Nonetheless, this praxeological organization is absent from the Indonesian textbook.

Another difference found in the point praxeology of graphing a function from Japan and Indonesia. In Japan, a function graph is firstly drawn within the domain of integers then progressively drawn within the domain of real numbers. Hence, it is clear that the graph is a straight line (for direct proportional and linear functions). However, in Indonesia, the function domain used is integers, which graph is theoretically depicted by points. Moreover, in the Japanese textbooks, the graph of $y = ax + b$ in grade 8 was developed from the graph of $y = ax$ in grade 7. This technique emphasizes the structural conception of a function, which Sfard (1991) defined beyond the output-input assignment, but to see a function as an object. Subsequently, the praxeological organization in Japan also exhibits meaning that the gradient of the graph (a) corresponds to the rate of change. In contrast, the y-intercept corresponds to the constant b in the function formula (Bardini & Stacey, 2006).

The most distinctive feature in learning function representation between Japan and Indonesia is the existence of function formula-and-graph connections in Japanese textbooks. Interpreting graphs as the structural conception of a function, Japanese textbooks implicitly explain that formulas exhibit both operational and structural conceptions (Sfard, 1991). In Japanese textbooks, the organizational approach indicates that functions are interconnected with other domains, such as geometry. Conversely, the textbook in Indonesia presents sets of ordered pairs as the form of a function, a praxeological organization absent in Japan because functions as sets have yet to be introduced, at least at this lower secondary level.

Although textbooks from both countries show various distinctions in their praxeology, both countries tend to use 'do something with the relation' techniques. As Mesa (2004) stated, these techniques are typically performed when the relation is already known. This study suggests that tasks requiring more 'do something about the relation' techniques (Mesa, 2004) could enhance learning experiences. As an illustration, making tables of values from the function formula given is considered 'do something with the relation', while making a table of values to build a function formula emphasizes 'do something about the relation'. Accordingly, tasks performed numerically, such as filling the table, are categorized as qualitative analyses of functions (Rolfes et al., 2018). On the other hand, the technique used to interpret the a and b in $y = ax + b$ from table, formula, and graph, which only appear in Japanese textbooks, are considered to analyze functions qualitatively (Rolfes et al., 2018). This technique is also helpful in building connections between function representations, constructing the idea that a function is a mathematical object, which entity holds the same across different forms (Bardini & Stacey, 2006; Doorman et al., 2012).

Finally, the differences and similarities observed can be further examined by the didactic co-determinacy of functions in both countries, elucidating from theme to domain level. The co-determinacy shows the conditions and constraints shaping function knowledge in each institution. As initial information, different terms are used to name mathematical domains between Japan and Indonesia. In Japan, the domains are classified into *number and algebraic expressions, geometrical figures, functions, and making use of data*. Meanwhile, in Indonesia, the domains are categorized into *number, algebra, measure, geometry, data analysis and probability*. Despite the different technical domain names used in both countries, this study described the domain using the widely-known terms in mathematics, *arithmetic, algebra, geometry, statistics and probability* to make the analysis easier. Thus, **Figure 4** shows the didactic co-determinacy of function knowledge in Japan and Indonesia.

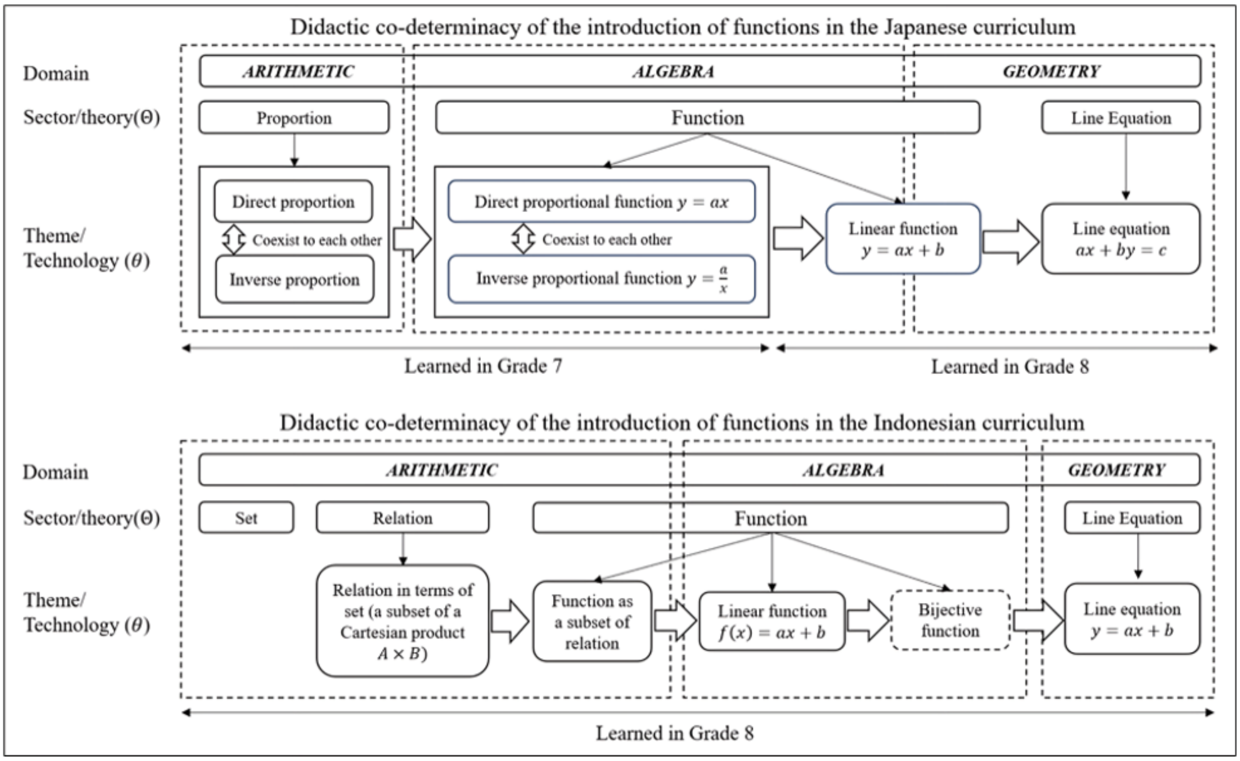

Figure 4. Level of didactic co-determinacy from domain to theme of functions in Japan and Indonesia (Source: Authors' own elaboration)

As illustrated in **Figure 4**, the initial stages of learning functions differ between Japan and Indonesia. In the Japanese curriculum, understanding the concept of function begins with proportion, which then leads to direct proportional functions represented as s $y = ax$. In line with Van Dooren et al. (2005), Japanese textbooks refer to $y = ax$ as a proportional function. Instead, they denote linear functions as $y = ax + b$. While learning direct and inverse proportional functions is considered to coexist, Japanese textbooks explain that inverse proportions cannot be developed into linear functions due to their non-additive rate of change (Confrey & Smith, 1994). Conversely, in the Indonesian curriculum, relations are the prerequisite knowledge for learning about functions, both defined by set theory.

Furthermore, the difference in the treatment of functions between Japan and Indonesia stems from disparate processes of didactic transposition within their respective educational systems, which is explained at the level of 'society' in didactic co-determinacy. Japan embraced the concept of functions before modern mathematics was introduced, while Indonesia integrated it very closely into the modern mathematics era. To grasp this disparity fully, an analysis of the post-modern mathematics curriculum in both countries is essential.

In Japan, post-modern mathematics reforms modified a new mathematical organization in the school curriculum that reinstated the original notion of functions, particularly emphasizing their application in dealing with quantities. Notably, recent educational initiatives in Japan advocate for a global reform centered on structured problem-solving methodologies, aligning mathematics more closely with real-world applications (Takahashi, 2006). This approach positions functions as entities representing changing quantities, facilitating a more meaningful connection between mathematics and practical problems, thereby transcending mere mathematical abstraction.

Conversely, while undergoing a 'back-to-basics' resurgence, the Indonesian post-modern mathematics curriculum continues to be influenced by traditional educational objectives emphasizing rote memorization and calculation skills (Mailizar et al., 2014). Formal definitions of mathematical concepts, including functions derived from set theory, are introduced early in the learning process, reflecting a preference for a structured, foundational approach to mathematics learning before applying them in real-life contexts.

Despite the distinctive features of functions knowledge in Japan and Indonesia, the dominant curriculum model in both countries established the same pattern: the knowledge tends to appear as a 'ready-made' topic, which can be used to solve sets of problems. According to Chevallard (2015), this perspective mainly fell into the *paradigm of visiting works*. Moreover, as Bosch (2018) stated that this traditional way of disseminating mathematical knowledge is inclined to have teacher-centered learning; constructing an understanding of the knowledge from answering questions provides a more epistemic way of learning new knowledge. To approach this issue, this future study will focus on an alternative reference epistemological model of function learning under the new *paradigm of questioning the world* and engaging the students through inquiry-based learning.

CONCLUSIONS

This study investigated and compared the learning of functions in Japanese and Indonesian textbooks. The analysis was carried out within the central ATD's construct, praxeology. Additionally, to gain deeper insights, this study adopted didactic co-determinacy to understand the conditions and constraints of functions topic within each educational system. This approach explained the similarities and differences uncovered through the praxeological analysis.

Regarding the *praxis* block, both Japanese and Indonesian textbooks feature task types centered around function definition and representation. However, notable differences emerge in the approaches. Japanese textbooks emphasize function formulas and graphs, whereas Indonesian textbooks predominantly present functions in sets of ordered pairs. Another difference was identified in that the textbooks in Japan present tasks related to inversely proportional functions, whereas Indonesia has tasks about bijective functions. Regarding techniques, Japanese textbooks offer a wider variety, encompassing both doing something 'with' and 'about' the relation. In contrast, the Indonesian textbook is dominated by techniques centered on doing something with the relation.

The theoretical discourse underpinning the *praxis* blocks in Japanese textbooks is based on functions as a rule. In contrast, in Indonesia, functions commence with the pairing between sets before transitioning into rules. Both countries adhere to distinct conditions for defining functions. In Japan, proportions serve as fundamental knowledge preceding the study of functions. Conversely, in Indonesia, sets and relations constitute the prerequisite knowledge for studying functions.

Further research could be extended on the taught knowledge process, reflecting the implemented function curriculum in both countries. Another essential study can also be extended to design the reference model for function learning, which engages students with inquiry-based learning.

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Ethical statement: The authors stated that the study does not require ethical approval, as it does not involve human or animal participants. The data utilized in this study were sourced from textbooks.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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