

Fostering Teacher Learning of Conjecturing, Generalising and Justifying through Mathematics Studio

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Calls to advance students' ability to engage in mathematical reasoning practices including conjecturing, generalising and justifying (CGJ) place significant new demands on teachers. This case study examines how Mathematics Studio provided opportunities for a team of U.S. middle school teachers to learn about these practices and ways to promote them in the classroom. Findings demonstrate how CGJ readings and focused discussions, coupled with repeated cycles of collaborative lesson planning, observation and debrief, supported the development of teacher knowledge, professional community, and teaching resources. In addition, this paper explores the role school leadership played in facilitating Math Studio to ensure these learning opportunities were realised. Documenting how Math Studio features and participants contributed to teachers' ability to implement CGJ focused lessons not only provides insights into the difficulties teachers have shifting instruction, but also adds to our understanding of school-embedded professional development more generally.

Keywords • professional development • lesson study • teacher knowledge • conjecturing, generalising, justifying

International efforts to reform mathematics have converged on the importance of engaging students in authentic mathematics practices that include conjecturing, generalising and justifying. In the United States this has been articulated in the Standards for Mathematical Practice component of the recently adopted Common Core State Standards (Common Core State Standards Initiative, 2010). These new standards call for students to make conjectures, construct viable arguments, and critique the reasoning of others, and thus represent a significant shift in both what and how mathematics is taught. Research documenting the difficulties teachers face when promoting these inquiry practices in the classroom (Goos, 2004; Staples, 2007), coupled with the persistence of traditional models of instruction (Jacobs, Hiebert, Givvin, Hollingsworth, Garnier, & Wearne, 2006; National Mathematics Advisory Panel, 2008) suggests that achieving this vision of instruction will require substantial teacher learning. Teachers need to not only learn more about conjecturing, generalising, and justifying, but also acquire pedagogical skills to foster these mathematical practices amongst their students.

Mathematics Studio, a school-based professional development model that incorporates several core components of lesson study (Fernandez & Yoshida, 2004), is a promising avenue for this new learning. In a typical Math Studio, mathematics teachers and administrators meet to collaboratively plan a lesson, record data during a live enactment of the lesson, and collectively reflect on the observed lesson. To investigate how this model can support changes in teacher practice, this paper draws on a yearlong study of Mathematics Studio implemented with a team of 7th grade teachers in a U.S. middle school. The goal of the Studio work was to improve teachers' ability to recognise mathematical conjectures, generalisations and justifications and design lessons that provide students opportunities to engage in these practices. Two interrelated research questions are explored: (1) What features of Mathematics Studio support teachers' learning in relation to promoting conjecturing, generalising and

justifying (CGJ) in the classroom? (2) What role do various participants, including the teachers, math coach, associate principal, and researcher, play in this learning process?

Why focus on Conjecturing, Generalising and Justifying

Conjecturing, generalising and justifying are fundamental practices in the discipline that play a vital role in the learning of mathematics (Ball & Bass, 2003; Yackel & Hanna, 2003). Participation in these practices supports students' abilities to think flexibly about mathematical ideas and relationships and to make sense of the mathematics they are learning (Carpenter, Franke, Levi, 2003; Lannin, Ellis & Elliott, 2011). Thus, a major goal of the Studio work was for teachers to develop shared understandings of CGJ, which we defined as follows:

Conjecturing: reasoning about mathematical relationships to develop statements that are tentatively thought to be true but are not known to be true (Lannin, Ellis & Elliott, 2011 p. 13).

Generalising: identifying commonalities across cases or extending mathematical reasoning beyond a single example, or set of examples to consider a broader range of objects (Ellis, 2011).

Justifying: developing a mathematically sound argument that uses disciplinary tools to demonstrate the truth or falsehood of a claim (Staples, 2014).

In light of studies documenting teachers' and students' fragile understandings of justification,¹ additional characteristics of CGJ were made explicit as they emerged during the Mathematics Studio (see Harel & Sowder, 2007; Lesseig, 2011, 2016). First, a mathematically sound justification is not based on authority, perception or consensus, but instead is a logical argument based on ideas previously accepted by the classroom community (Lannin, Ellis & Elliott, 2011; Stylianides, 2007). Secondly, although a counterexample is a valid form of refutation, testing examples is insufficient to justify the truth of a general statement. However, examples do play an important role in formulating conjectures, testing the boundaries of generalisations, and uncovering structure that can later be used in creating the justification (Lannin, Barker, Townsend, 2006; Pedemonte & Buchbinder, 2011; Zazkis, Liljedahl & Chernoff, 2008). These characteristics of CGJ were made explicit as they emerged during the Mathematics Studio work.

Recognizing the important role CGJ play in learning mathematics, researchers have investigated students' conjecturing, generalising, and justifying behaviour and the instructional strategies that support or inhibit students' participation in these practices (e.g. Ellis, 2011; Lampert, 2001; Lannin, 2005; Martino & Maher, 1999; Reid, 2002; Staples, 2007). These studies point to the importance of task design, collaborative work, and teachers' questioning strategies. For example, Ellis (2011) identified ways in which generalising behaviour was promoted through specific requests to publicly justify or clarify, or by asking questions that encouraged students to relate two or more entities or extend beyond the case at hand. Classroom images garnered from this work demonstrate that students at all levels are capable of CGJ. However, creating opportunities for them to do so is no easy task and will require substantive teacher learning.

Teacher Learning Communities

The epistemological stance underlying Mathematics Studio is that professional development occurring within teacher communities investigating the day-to-day work of teaching can support knowledge development and changes in teaching practice (Cochran-Smith & Lytle, 1999; Lave & Wenger, 1999; Little, 2002). There is mounting evidence that teacher learning

¹ Although justification, as used in this study, was held to standards often reserved for mathematical proof, the term justification was used throughout the study because it was more approachable to teachers at the middle grades.

communities can have a positive impact on both teaching practice and student achievement (Vescio, Ross & Adams, 2008). However, merely providing time for teachers to work together is not sufficient (DuFour, 2004; Supovitz & Christman, 2003). To make substantial instructional changes, teachers need ongoing opportunities to learn through collaborative discussions that focus on student learning (Stoll, Bolam, McMahon, Wallace & Thomas, 2006; Supovitz's, 2002; Vescio, Ross & Adams, 2008).

Two ideas of note from the research on teacher learning communities are the importance of structured and supported conversations and the consistent focus on examining relationships between instruction and student work. To date, little attention has been paid to how facilitation moves, protocols, and norms work together to enable this focus. Despite the abundance of available protocols to guide professional learning communities (e.g. Easton, 2009; Fogarty & Pete, 2010) studies on the use of protocols and structured conversations to enhance teacher learning have produced mixed results (DuFour, Eaker & Many, 2010; Ermeling, 2010; Little & Curry, 2009). Without skilled facilitation, strict adherence to superficial features of the protocol (e.g. providing equal sharing time, rotating facilitator roles) can limit opportunities for teachers to critically investigate teaching and learning dilemmas (Little & Curry, 2009). That is, while protocol-based conversations have the potential to elicit broad participation and build community, the protocol alone is insufficient to ensure teacher learning.

The Promise of Lesson Study

With a focus on investigating instruction through collaboratively designing and observing lessons, lesson study is a promising structure to foster a productive teacher learning community. Lesson study, the dominant form of professional development in Japan, has undergone a variety of necessary adaptations as it has spread throughout the U.S. and internationally (Perry & Lewis, 2009; Robinson & Leikin, 2012). Despite variation in size and scope, the central feature of any lesson study cycle is the observation of live classroom lessons. The classroom lesson provides a context for teacher groups to collect data and analyse relationships between instruction and student learning (Yoshida, 2008; Hart, Alston, & Murata 2011). Case study data reveals how lesson study can support the development of knowledge, skills, and dispositions necessary for changing instructional practice (Hart, Alston & Murata, 2011; Hunter & Back, 2011; Lewis, 2009; Perry & Lewis, 2009). Specifically, researchers have documented ways in which lesson study enhances teachers' knowledge of content, pedagogy, and student thinking (e.g. Alston, Pedrick, Morris & Basu, 2011; Fernandez, 2005; Tepylo & Moss, 2011) as well as the ability to link these three aspects of practice in productive ways (Murata, Bofferding, Pothén, Taylor & Wischnia, 2012).

However, as with teacher communities in general, there is strong evidence that the lesson study facilitator and other experts play a key role in the learning opportunities made available (Murata, et al., 2012; Yoshida, 2008). In their investigation of a lesson study collaboration amongst 5th and 6th grade teachers and Japanese educators, Fernandez, Cannon and Chokshi (2003) documented ways in which the Japanese coaches gave advice to U.S. lesson study participants. Of note was how the Japanese coaches made repeated connections to broader principles of instruction and attempted to focus teachers' examination of lessons through three critical lenses (researcher, curricular and student). Fernandez and colleagues suggest that in order to make lesson study a powerful experience, U.S. teachers will need similar outside support to develop these critical lenses and move beyond traditional instructional practices.

Case study data revealing the positive benefits of teacher learning communities and of lesson study in particular are mounting. However, efforts to scale-up these localised efforts are dependent on the identification of underlying principles. In other words, more research is needed on *how* lesson study supports teacher learning and leads to instructional improvements. Based on their investigation of elementary lesson study teams, Robinson and Leikin (2012) advance three mechanisms that led to changes in teachers' instructional practices: collaborative noticing, collaborative awareness, and brainstorming. Lewis, Perry and Hurd (2009) similarly posit that the collaborative nature of the process, making teacher beliefs and knowledge visible and available for discussion, is a critical aspect toward changing instruction.

The theoretical model developed by Lewis, Perry and Hurd (2009, p. 287) provides a comprehensive view of how lesson study can lead to improvements in instruction and student learning through changes in teachers' knowledge and beliefs, professional community, and in teaching-learning resources. First, they posit that through lesson study, colleagues' ideas about content, pedagogy, and student thinking are made visible, enabling teachers to develop or refine their knowledge for teaching. Second, lesson study strengthens professional community through the development of collaborative norms, shared frameworks or tools to analyse practice, and mutual accountability to provide high-quality instruction. Finally, instructional change and student learning are supported through the development of teaching resources such as lesson plans or common tasks and assessments. These change mechanisms provided a useful framework to characterise teacher learning in Mathematics Studio.

My study builds on this work to consider how Mathematics Studio supports the development of teacher knowledge, professional community and resources necessary to promote conjecturing, generalisation and justification in mathematics classrooms.

Methods

Participants and Context

During the 2012-2013 academic year, the author and math coach led six Mathematics Studio cycles with the 7th grade math teachers and associate principal. As summarised in table 1 below, teacher participants had a range of teaching experience and mathematics preparation. All four teachers held a Masters in Teaching degree, but only two, Beth and Sean, had specific endorsements in mathematics. Prior to the start of the first Mathematics Studio, Gary was chosen as the Studio teacher.² In consultation with the researcher and associate principal, the math coach made this selection based on current teaching practices and perceived openness to the process. Gary had the most experience teaching at this grade level and still held tightly to many traditional teaching methods. However, Gary had expressed a desire to make changes and was willing to have others observe his classroom. These qualities made Gary an ideal case because his practice was approachable by others in the group, as opposed to offering images of "idealised" instruction, and thus afforded a true learning environment.

The associate principal, Sara, had over 12 years of administrative experience. Sara had taught 3-5 years at both the elementary and middle school levels but had no formal training in mathematics. However, she had been intimately involved in the mathematics work at the district level for several years and was charged with supervising and evaluating all mathematics teachers. Pam, the mathematics coach had taught at the middle and high school level for approximately 20 years and had served as a district level math coach for four years prior to this study.

My own role in the study is best defined as that of participant observer (DeWalt & DeWalt, 2010). While I played a key role in planning each Studio Cycle, I interacted as an equal participant in the discussions that occurred during the Studio Days. I made a conscious effort to neither withhold my ideas, nor to steer discussions toward a fixed end. At the same time I recognise that I was often positioned as the expert given my University status, previous experience with Mathematics Studio and research focus on mathematical justification and teacher knowledge of proof (Lesseig, 2016).

² In some cases groups may rotate the Studio teacher, however given the dynamics of this teacher group and our desire to follow one group of students we felt that it would be most productive to remain in one classroom for the year.

Table 1
Participant teaching experience and credentials

Participant	Years with 7 th grade team	Other teaching experience	Teaching certification
Gary Studio teacher	10+	7 th & 8 th grade science, 3-5 years	K-8 Masters in Teaching (elementary)
Beth Teacher	6	3-5 th grade, 3-5 years	K-8 Masters in Teaching (elementary) Middle Level Math Endorsement
Sue Teacher	1	6 th grade, 10+ years	K-8 Masters in Teaching (elementary)
Sean Teacher	0	None	Mathematics grades 5-12 Masters in Teaching (secondary)
Sara Associate principal	4	3-5 th grade, 3-5 years 6-8 th grade, 3-5 years	K-8 Masters in Teaching (elementary) Principal Certification
Pam Math coach	4	K-5, less than 3 6-8, 10+ years 9-12, 10+ years	K-8; Mathematics 9-12 Middle Level Math & Science Endorsement
Kristin Researcher / Mathematics teacher educator	0	6 th grade, 5 years 9-12, 9 years University, 5-10 years	Secondary Math and Science certification PhD in Mathematics Education

The Math Studio work was designed to deepen teachers' understanding of conjecturing, generalising and justifying (CGJ) so they could actively promote these practices in the classroom. To support this goal, the 2-day Studio cycle typically began with a discussion of CGJ supported by outside readings. Teachers then spent the remainder of the first day co-designing a lesson to elicit CGJ. Each lesson drew on the district curricula and was designed to encompass the mathematical content currently being addressed in the Studio teacher's class.

Two days later, teachers observed the Studio teacher implementing the lesson in his first period class. Teachers debriefed the lesson by first categorising student discourse they had recorded as a procedure or fact, conjecture, generalisation, or justification. Using this data, teachers drew conclusions about students' ability to conjecture, generalise and justify and students' understanding of specific mathematical concepts targeted in the lesson. Next, teachers critically reviewed the lesson, as written and implemented, to identify elements that supported or hindered opportunities for CGJ. The Studio ended with participants reflecting on what they had learned and how they might adjust their own practice as a result. Between Studio cycles teachers were encouraged to implement the Studio lesson (or revised version of the lesson) with their own students and share the results during the next Math Studio. The foci of each Mathematics Studio Cycle activity are illustrated in Figure 1.

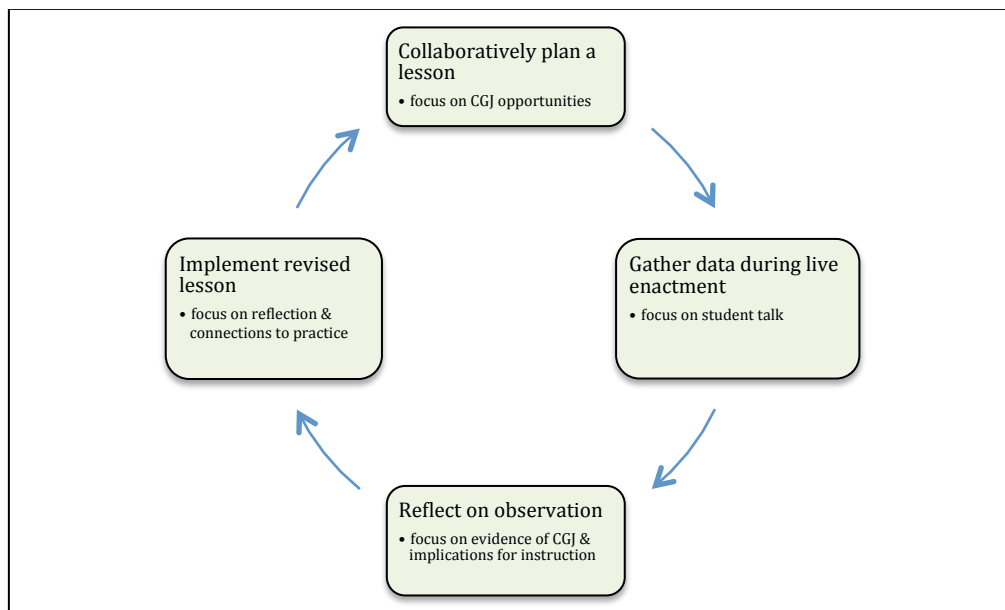


Figure 1: Mathematics Studio cycle

Data and Analysis

Consistent with case study methodology (Yin, 2009) and the underlying perspective that teacher learning occurs through ongoing interactions amongst colleagues and artefacts of teaching and learning (Little, 2002), multiple sources of evidence were drawn upon to make sense of the learning opportunities that arose during Math Studio (research question one) and the ways in which various participants contributed to their fruition (research question two). Data collected included video, field notes and artefacts from each Math Studio session, artefacts from mid-cycle observations and planning meetings, and questionnaires participants completed at the beginning, middle and end of the year.

Studiocode© (Studiocode Business Group, 2012), a qualitative video analysis software, was used to analyse all video data. A timeline was created for each of the six Math Studio sessions and video segments first chunked into idea units. Idea units were defined according to topic shifts and natural breaks in a discussion.³ Through multiple analytic passes, these idea units were coded according to Studio activity (e.g. readings, lesson planning or debrief), speaker and topic. Topic codes included a primary code of *pedagogy*, *math content*, *students*, or *CGJ* to denote the main focus of the discussion as well as secondary codes such as *classroom example*, *purpose*, or *challenge* to further detail the conversation. After this initial, structural coding (Saldana, 2012), change mechanism codes were added to capture moments when teacher knowledge and beliefs, professional community or teaching-learning resources (Lewis, Perry & Hurd, 2009) were potentially developed or drawn upon.

These coded timelines were then combined into a single database. The data matrix and Boolean search features within Studiocode were used to calculate code frequencies and identify overall patterns (i.e. connections between speaker or activity and change mechanism) and support further analysis. For example, all idea units coded as both *pedagogy* and *CGJ* or *pedagogy* and *purpose* could be combined and viewed in succession in order to identify practices teachers developed to promote mathematical discourse, or ways in which the rationales for pedagogical decisions were made explicit in the group.

³ In order to preserve the integrity of the conversation thread, I did not code turns of talk. Thus, idea units varied in length, included multiple topic codes and might involve one or more speakers.

Video analysis was coordinated with reviews of teacher reflections and artefacts from each Math Studio cycle including Studio agendas, readings, and lesson plans the group developed. Throughout this process, the construction of initial and integrative memos (Emerson, Fretz & Shaw, 1995) afforded comparisons across Studio cycles to identify themes in teachers' activity and trace developing ideas about designing lessons that promote CGJ.

In the findings section below, I begin with examples of teacher learning evidenced across each of the Math Studio activities to address research question one: What features of Mathematics Studio support teachers' learning in relation to promoting conjecturing, generalising and justifying (CGJ) in the classroom? This is followed by further elaboration on how participants, together with Studio tools and prompts, contributed to these learning opportunities, research question two.

Linking Math Studio Activity to Teacher Learning Opportunities

Similar to other lesson study researchers, I found clear links between Mathematics Studio activity and the three mechanisms posited to support changes in teacher practice: 1) advances in teacher knowledge and beliefs; 2) strengthening of professional community; and 3) development of teaching resources (Lewis, Perry & Hurd, 2009). Although the distinctions between these avenues for change are not always clear-cut, I have nonetheless organised findings in that way to facilitate comparisons and make connections to the second research question. These connections are also summarised in Table 2.

Advancing Content and Pedagogical Content Knowledge

CGJ readings and subsequent discussions infused new ideas about CGJ. Excerpts from *Developing Essential Understanding of Mathematical Reasoning* (Lannin, Ellis & Elliott, 2011) provided an initial springboard into CGJ conversations. In the first Studio, teachers generated a CGJ matrix to capture the group's evolving definitions, examples and questions about these mathematical practices. This tool became a valuable resource that was revisited and refined throughout the remaining Studio cycles as the group continued to grapple with the concepts and had further experience designing and observing lessons. Coupled with lesson observations, these outside resources helped the group develop more nuanced understandings of conjecturing, generalising and justifying and ways to support these practices in the classroom.

For example, when considering characteristics of a valid justification, teachers initially stated that justification must use accepted math truths and follow a logical progression. Later teachers also considered the role of audience and discussed how students' justification could be represented with numbers, pictures, or a verbal explanation. Lingering questions such as, "what constitutes enough justification?" or "how do we create the need to prove something?" helped teachers connect their understandings of mathematical justification to their work with students. Teachers recognised that they needed to explicitly ask for justification, model "good justifications," and provide opportunities for students to critique justifications offered by their peers. Based on observations that students did not always feel compelled to provide a justification and often gave incomplete justifications, teachers discussed ways to motivate justification by having students generate their own questions or by creating a foil to confront

students' previous conceptions. Such discussions are particularly powerful in light of research documenting the importance of creating an intellectual need to move beyond authoritarian or empirical justifications toward more robust forms of proof (Harel & Sowder, 2007; Stylianides & Stylianides, 2009).

The collaborative lesson planning process highlighted the intentional planning needed to promote CGJ and made teachers mathematical and pedagogical thinking visible. Teachers had to brainstorm and sift through ideas quickly to develop a plan that could be enacted in two days. Thus, they were continually pressed to defend pedagogical choices based on the potential to elicit student conjectures, generalisations and justifications and to articulate underlying math ideas or generalisations they wanted students to develop by the end of the lesson.

Table 2
Connections between Studio activities and learning opportunities

Change Mechanism	Math studio feature	Key move or structure
Teacher Knowledge - increased understanding of CGJ and strategies to promote these practices in the classroom	Readings and CGJ discussions	CGJ matrix and public records to document evolving understandings and questions
	Lesson planning	Articulating lesson goal and big ideas Pressing for pedagogical rationale Reiterating purpose of Math Studio
	Lesson observation	Tool to record and categorise student discourse
	Lesson debrief	Pressing for evidence of CGJ in student talk or action Analyzing task in light of student thinking
	Multiple 2-day cycles throughout year	Maintaining CGJ focus and coherence across studio sessions
Professional Community - shared beliefs and commitments to implement teaching practices that promote CGJ - shared vision of practice	Opening and closing reflections	Publicly sharing individual next steps and results from classroom implementations
	Collaborative lesson planning	Participation structures and norms Brainstorming and public recording of ideas
	Lesson observation & debrief	Naming instructional moves and structures Connecting task to specific student activity
Teaching and Learning Resources - modified tasks, lesson plans and generalisable strategies that promote CGJ	Administrative participation	Aligning with building goals Reiterating Math Studio purpose
	Collaborative lesson planning	Compiling materials, handouts, lesson plans
	Lesson observation	Developing protocols for investigating practice
	Lesson debrief, revisions	Documenting pedagogical strategies

Teachers also adopted a CGJ lens to weigh the affordances and constraints of different student groupings or task features (e.g. what numbers to provide or whether to allow calculators). For example, in Studio 5, the group spent considerable time debating whether the problem used to launch the lesson should come out evenly. The task posed was to compute the number of months required to purchase a \$2500 car with monthly payments of \$250. The discussion revolved around whether the down payment should be \$500 or \$400 (which unlike \$500 might require more difficult computations and result in a fractional answer). The group eventually agreed that having a whole number answer would encourage efficient mental strategies and

shift the focus away from trying to compute the correct answer, toward articulating a process that could then be connected to equations.

Finally, the common lesson observations promoted a focus on student talk. When asked how Mathematics Studio contributed to their own learning, teachers overwhelmingly ranked co-planning and observing the lesson as the most valuable aspects. Observing a colleague teach the lesson allowed teachers to focus on the student-teacher interactions without the burden of responding in the moment or managing the classroom. Thus, teachers were able to “see the flow and types of questions asked” and attend to student thinking more deeply. In his exit reflection from Studio 5, Sean commented that through this process, he kept learning better ways to respond to student questions with other questions, and to “probe their thoughts, rather than giving answers right away.” Similarly, Beth wrote that she appreciated watching and listening to students doing math. “It has helped me be more cognizant of how I talk to students about their thinking. Less is more. Don't say anything a kid could say.”

In sum, multiple activities contributed to building teachers' content and pedagogical content knowledge related to CGJ. The collaborative nature of the Mathematics Studio work made this knowledge visible and available to the group. Perhaps the most important take-away for teachers was an awareness of the need to intentionally plan for CGJ. In the final survey, Sean captured this sentiment in his description of how Mathematics Studio impacted the way he now approaches instruction,

I am now more careful to anticipate what students are going to think, say, and do every day. I am also more careful about planning for disagreements and discussion opportunities throughout the daily activities that will allow students to make conjectures, generalisations, and especially to justify their reasoning to their peers.

Strengthening Professional Community

In addition to building knowledge of content, pedagogy, and student thinking, Mathematics Studio helped teachers create a shared vision of what classroom instruction focused on student generated conjectures, generalisations and justifications might look like. This vision was reflected in the development of a shared language and commitments to change instructional practices.

The Mathematics Studio readings and lesson debriefs provided teachers with images of student-centred instruction and language to describe specific pedagogical strategies to foster CGJ. Teachers talked about the importance of asking “high press questions” (Kazemi, 1998) creating a “foil”, or using “four corners,” to promote justification. Adopted phrases such as “less is more,” were indicative of the group's joint recognition that to move beyond procedures and facts, students needed fewer problems.

Opportunities for teachers to openly reflect on their own practice contributed to this shared vision of instruction and made individual teacher's commitments to change public. Together with explicit messages that the lessons we were planning needed to be owned by the group, the expectation to share results from their own implementation of the Studio lesson kept teachers accountable. As discussed later, the establishment of a learning community committed to changing practice was supported by the associate principal's active participation in Mathematics Studio. This administrative presence reinforced the fact that the instruction promoted in Mathematics Studio was tied to building-level initiatives to engage all students in productive struggle and increase the cognitive demand of lessons. Teachers thus received clear messages that they were responsible for making changes, and that they would be supported in those efforts.

Developing Teaching and Learning Resources

Specific resources generated through Mathematics Studio included the CGJ matrix as well as teaching materials (e.g. lesson plans and student worksheets) that the math coach took upon herself to revise, copy and make available. Other less tangible resources included: 1) pedagogical strategies to elicit CGJ that could be named and shared amongst the group; and 2) protocols for observing & reflecting on their own and their colleagues' instruction. These

resources were a direct consequence of the collaborative lesson planning, observing and debriefing process.

Teachers naturally developed a shared language to describe the lesson structures they planned and implemented (Staples & Truxaw, 2007; Staples, 2008). In her survey response, Beth echoed the importance of documenting and making these resources available.

I want structures or protocols that other teachers use. Some things I call structures (or protocols) include building consensus, four corners, our Field Goal Hazzah debrief with gestures... We need to name these things. What I really want is for our department to come up with a menu of protocols and resources so we get some consistency and build these things in all of our students from one class to the next.

Prior to Math Studio, observing a colleague teach a lesson was not something teachers were accustomed to doing. Thus it was important to establish clear guidelines and provide a standard observational tool for teachers to record verbatim what they heard students say and do during the lesson. This intentional focus on student discourse, as opposed to teacher talk and actions, created a safe space for investigation. More importantly, the protocol directed teachers' attention to characteristics of CGJ elicited from students and could be used to frame later observations in their own and their colleagues' classrooms.

Participant Contributions to Learning Opportunities

Researchers have suggested that successful lesson study requires skilled facilitation to take full advantage of teacher learning opportunities (Fernandez, 2005; Lewis, Perry & Hurd, 2009). Similarly, my data revealed ways that both the math coach and researcher played a key role by making mathematical and pedagogical ideas explicit and reiterating the purpose of Mathematics Studio. My study also indicated ways in which the associate principal and other teachers contributed by asking genuine questions about the mathematics or student thinking and by defending pedagogical choices. This paper focuses on the specific contributions of the math coach and associate principal. Understanding the role of these school leaders in particular is critical to the sustainability and success of Math Studio and school embedded professional development more generally. Because their influence was most apparent in relation to promoting teacher learning and building professional community, the findings presented concentrate on those two areas.

Mathematics Coach and Associate Principal Roles in Promoting Teacher Learning

One of the distinctive features of Mathematics Studio is the integral role of school leadership. While I had a significant influence on the overarching structure and focus of the Mathematics Studio work, the math coach, Pam, was primarily responsible for facilitating the sessions with teachers. We met frequently to plan and debrief each Studio cycle and discuss learning goals for teachers based on data from classroom observations and teacher reflections. Sara, the associate principal also met with us between cycles to observe classrooms and provide her perspective on teachers' current practices. On Studio days, both Sara and Pam fluidly moved between leadership and participant roles, sending explicit and implicit messages about how and why the group should engage in this collaborative work. In my analysis of specific coach and associate principal moves, three categories emerged as fundamental to advancing the group's learning: 1) making teachers' thinking public, 2) challenging mathematics learning goals and 3) requesting a rationale for pedagogical decisions.

First, both the associate principal and math coach made sure that teacher thinking was public and open for debate. The purposeful planning for each Mathematics Studio session resulted in detailed agendas, discussion prompts and readings designed to stimulate new thinking. However, to make sure these learning opportunities were realised, the math coach had to continually monitor and record teachers' contributions. Suggestions for the student task and grouping structures were recorded, elaborated and sometimes erased as the group planned

the lesson. Throughout this brainstorming process, teachers were asked to defend choices and come to consensus based on potential to elicit student conjectures, generalisations, or justifications. As a result, the discussions did not devolve to simply sharing and voting on the best idea, as is sometimes the case when teachers work collaboratively (Little & Curry, 2009).

This process of recording and defending thinking was also evident during the lesson debrief when teachers shared examples of student discourse and categorised the student talk as procedures and facts, conjecture, generalisation, or justification. As the examples were posted and sorted, Pam pressed teachers to defend their categorisations and elicited alternative interpretations. Teachers were thus compelled to revisit the characteristics of CGJ. Debates over whether a student was making a conjecture or whether a student's justification was complete increased teachers understanding of CGJ and laid the foundation for further conversations about how to move students toward generalisations or more sophisticated mathematical arguments.

Second, the math coach and associate principal challenged teachers to connect the lesson goals to big mathematical ideas or generalisations. During the Studio day, Sara worked on the math task and offered lesson suggestions alongside the 7th grade math teachers. But, by positioning herself as "not the math person," she was able to ask genuine questions about the mathematics and why a particular topic such as indirect measurement was relevant. To reach consensus on the learning target teachers needed to convince Sara that the mathematical ideas in the lesson were compelling. This forced teachers to think deeply about the underlying concepts and design lessons that moved beyond procedures and facts toward more powerful generalisations and justifications.

The excerpt from Studio 5 below illustrates one such exchange. The Studio teacher had identified "using equations to represent and solve problems involving linear relationships" as the lesson topic. The group decided to begin the lesson by showing a picture of a car and asking students to determine how many months it would take to purchase the car given the down payment and monthly instalment. After working on the task themselves, Sara pressed the group to explain why writing the equation is important.

- Sara: Feel free to take what I say with a grain of salt, but I'm still hung up on the fact, do kids understand why they have to do equations? What is the big reason why kids need to do an equation? Is it because the numbers will get too difficult? And frankly, I would go around equations to do that myself. So I am wondering, if the target is getting kids to be able to create an equation from a context...if they don't get why they do an equation to begin with... Maybe this is not a relevant question, but if we don't understand why we use equations, and this task doesn't make me use one...
- Beth: My argument is that you are doing equations. You just did it in your head (referring to how Sara had originally solved the problem) so I am showing you a symbolic way.
- Sara: So would it be appropriate to say to the kids, this one you could probably solve without the symbolic ways, but what if the mathematics was a lot more complex?

The discussion continued with the group coming to consensus about structuring the work time so students could investigate a series of problems in which the parameters are changed (e.g. initial down payment or monthly payment). This would set the stage for students to consider how and why equations are important.

- Sara: Because I think like a 7th or 8th grader when it comes to math, this makes sense to me. Keeping the situation the same and changing one piece, so students can learn what I just learned - that every time I don't have to do the whole thing, I only have to change one piece. That's why we use equations.

Having to defend the importance of equations forced the group to reconsider their standard textbook problem for writing and solving linear equations and led them to redesign the task so students could analyse the effects of changing parameters and make generalisations about the utility of equations.

The third category of moves involved challenging and making teachers' pedagogical reasoning explicit. Unlike in Japan, U.S. teachers' manuals seldom include explicit learning goals or a rationale for lesson design (Lewis, Perry, Friedkin & Roth, 2012). Thus, instances when teachers included the purpose behind lesson suggestions seemed particularly productive. My analysis indicated that while teachers sometimes offered a rationale or potential affordance when offering a strategy or change to the lesson, most often these exchanges were prompted by either the math coach or associate principal. For example, if teachers did not automatically state why a task should be implemented a particular way, Pam or Sara would follow-up by asking, "What would we gain by having them work with partners first?" or simply, "and why would you have students do that?" These moves pressed teachers to think about how different structures and tasks promote student engagement in CGJ and established norms that we needed to plan intentionally.

In addition to asking teachers to defend pedagogical decisions, Sara pressed teachers to name aspects of instruction that supported particular student outcomes. For example in the final Studio session, when Sean was reflecting on his implementation of the previous lesson, Sara questioned Sean about connections between the task and student engagement. Recall the lesson from Studio 5 was designed to support students' generalising and justifying about equations in the context of figuring out how long it would take to buy a car on a payment plan. Sean expressed that he was pleased with the overall lesson and by what his students seemed to understand.

- Pam: What surprised you?
- Sean: I felt there was an increase in effort. I think it was the way they were grouped, the ABCD so that they each had their own numbers but could talk to each other and could almost piggy back on each other. Usually I get most students involved, but to get all of them to give me something was unusual.
- Sara: So what was it about that task that made everybody jump in?
- Pam: Well he was saying it was about the assignment of the different parts
- Sara: Can you even come up a little higher and describe what that was, if we were talking about intentional task design, what would you call it?

Moves by both the associate principal and math coach to make pedagogical rationales explicit not only supported the development of teacher knowledge but also promoted professional community as described next.

Mathematics Coach and Principal Roles in Promoting Professional Community

Based on early discussions with the coach and associate principal, it was clear that we shared a common vision of instruction that involved less teacher direction and was focused on student reasoning and sense-making through mathematical discourse. The associate principal and district math coach both sought to move all teachers along in their practice and saw how the Mathematics Studio goal of encouraging student conjecturing, generalising and justifying was clearly aligned with the overall building problem of practice, "to engage all students in productive struggle." Throughout the Mathematics Studio work, Sara and Pam worked in subtle and sometimes very explicit ways to advance this agenda by reiterating the purpose of teachers working together and connecting this to other building initiatives.

First and foremost, Pam and Sara supported shared goals and maintained a focus on CGJ by explicitly stating the purpose of each Studio activity. When giving directions to move from one activity to the next, Pam would clarify both the task at hand and the reporting strategy or protocol we would use to share or record ideas. Mindful of dominant voices in the group, Pam used several structures to ensure equal participation and solicit everyone's ideas. Beyond these generic facilitation moves to elicit and record everyone's thinking, Pam constantly reiterated the purpose of working collaboratively and our overarching Studio goal to support students' ability to CGJ. Pam's statement during the lesson-planning phase of Studio 2 is a typical example:

We all benefit from the process of working together planning a lesson. And we need to be able to teach it in our classroom and bring feedback because even if we plan something together, it will play out differently in everybody's classroom. So the first thing is to make sure our target is narrow enough that we can accomplish it in a class period. The other thing is to go back and revisit *our* learning target --where are there opportunities in this lesson for students to participate in discourse and to come up with some conjectures, generalisations and justifications?

In this way the coach not only promoted buy-in but also reinforced the idea that if the lesson was to be owned by the group we all needed to come to consensus on the underlying mathematical and pedagogical ideas. Pam stressed that the resulting lesson needed to be one that everyone felt good about and could say, "I understand it, I understand the math, I understand the outcomes and I will know whether or not I hit those targets."

Second, Sara was quite transparent with teachers about how the Studio focus on CGJ fit with overall building initiatives (i.e. promoting student discourse through the workshop model and implementation of high cognitive demand tasks). As evidenced in the detailed agenda (see Appendix), we revised the lesson planning prompts to better align with the workshop model adopted by the district. Pam and Sara continued to reinforce this instructional model when asking teachers to elaborate on the purpose of the debrief or to share what a typical launch looked like in their classrooms. In addition to subtle reinforcements, other more pointed conversations also occurred. For example, during the mid-cycle rounds and pre-planning conversations, Sara had noted that debriefs weren't happening as intended in math classes. Instead of drawing on student ideas, teachers were typically using debrief time for additional teaching. So, in the midst of planning the lesson in Studio 4, Sara followed up on Gary's statement about what he wanted to summarise at the end of the lesson to press on the purpose of a lesson debrief. In the lesson, students were going to be given price scenarios from a variety of stores and asked to determine the best buy for several common items.

Sara: I just want to direct our attention to the Workshop wheel (pointing to diagram posted in the room) and what debrief is for. It's for *students* to share understanding and thinking and the teacher labels or holds this thinking. Debrief is not where we make those generalisations for them. It's where they say and we notice what they say.

In defence, Gary tried to clarify how he envisioned orchestrating this summarising discussion. Note however that Sara does not stop here but continues to press for some public recording of student ideas:

Gary: I'm just setting it up, I'm giving them a window to look through and then I'm asking the question and they're telling me what they understood.

Sara: What's your question going to be?

Gary: Which one is the better deal and how do you know? And hopefully they will say, we are talking about quantity here so...

Sara: How are you going to capture what they say? Or are you?

Gary: It's just going to be in a group share

Sara: I know but if somebody comes up with this phrase, this great generalisation, or a justification for why

Gary: Well, everybody puts it in their notes, and we vote too.

Sara: I am just wondering if it's good to hold on to that like an anchor chart. Right now this is what we believe, our generalisation...

From here teachers continued to brainstorm ways to record student thinking and generated a list of strategies that Gary might look for to share, label and record. Potential prompts for Gary to use in the debrief discussion such as, "what are you noticing about all those strategies?" were recorded and became shared resources within the group. This routine of generating

potential discussion prompts or back-pocket questions was carried through in the remaining Studio lessons.

A second example of making connections to building initiatives occurred in the final Studio session. During the lesson debrief, Sara introduced teachers to the Hess Matrix, which integrates Bloom's taxonomy with Webb's Depth of Knowledge component (Hess, Jones, Carlock & Walkup, 2009). Sara first shared how the tool helped her learn that while not every task lends itself to the analysis or synthesis levels, there were still ways to press for deeper understanding. Below is an excerpt from the conversation that ensued after reviewing the matrix. This summarising discussion illustrates the group's collective thinking by the end of the year, and ways in which the associate principal and teachers were able to link their Math Studio experiences to classroom instruction.

- Sara: So if you look at the 'understand' in Bloom's then read to the right under Webb's level one, two, three, just read to the right and see how a task might change. This helped me to understand how we can press for that deeper understanding. Down and to the right is what we are moving toward.
- Sean: What I notice when I look at this is the word generalise coming out more and more as we move to the right.
- Kristin: I am looking at level 3 and I see, 'explain, generalise or connect ideas using supporting evidence' and 'make conjectures and justify them.'
- Sara: So what we have been working on in Studio all year lives over here in the level 3 and maybe even in the level 4.
- Gary: It's how you present it.
- Sara: It can be, but also to help us understand it's okay for our learning targets to sometimes be at the lower level but moving them to the right by... yea, by building your presentation differently like we did today.
- Gary: You can present it at level 3, or 4 then after kids understand you can give them the one-step, two-step worksheet where they are applying. But I sometimes teach in reverse, I will assume that they have to do these easy - 'I'm going to teach you how to do it, here's how you do it' - and then I try to go back and I try to extend to get them to understand why it works. I think I need to go backwards.

Teachers saw how the practices of CGJ were associated with higher depths of knowledge and were able to make connections to things we had worked on in Math Studio. Gary's statements about teaching backwards, to build understanding before introducing specific procedures, was a significant shift from his concerns in early Studio sessions about providing students with "muddy water tasks." As the discussion continued, Beth described her own practice and opened the door for others to reflect on the benefits of providing students with rich, open-ended problems before teaching procedures.

- Beth: I always do it backwards.
- Sara: So go for understanding first, then go back
- Beth: I give these really open-ended things and a lot of times I waste a lot of time because sometimes it's all over the place, but sometimes it's not. But you have to be able to be okay with the chaos.
- Kristin: But even the chaos today, you saw all that rich mathematics and it came from the students.
- Sara: And we would never have gotten the 6/8ths, the .75 (referring to specific math ideas that emerged during the Studio lesson from students who had used a scale factor approach to solve the problem). We would have never gotten that if we would have said, just do it this way.

Sara later concluded the conversation by encouraging teachers to use this depth of knowledge matrix next year as a tool to plan lessons that lie further to the right by “building in the task, like Gary said, by creating the task that allows students to do those things and to get at that reasoning we are trying for.”

This final Studio discussion highlights how common lesson planning and observation provided a venue for teachers to try out ambitious lessons and investigate the effects. In this case, teachers saw how the open-ended design of the task increased student engagement and created opportunities for students to defend multiple approaches.

Discussion

The primary purpose of this paper was to identify how Mathematics Studio provided opportunities for teachers to develop knowledge, skills and resources to engage students in CGJ. Given that these practices are not commonplace, and that teachers admittedly had a lot to learn, it makes sense that all of the activities within a Studio cycle proved to be productive. The readings not only prompted questions about instruction but also introduced mathematically precise definitions of CGJ—new content for many teachers. Collaborative lesson planning supported teachers in intentionally planning for CGJ (also a new experience) and brainstorming strategies to engage students in those practices. During the lesson observations, teachers had a unique opportunity to listen to students (without the distraction of managing the class) and look for CGJ behaviours. Finally, during the debrief teachers could collectively make sense of student discourse in relation to CGJ. Big challenges for teachers regarding CGJ are identifying similarities and differences among the component practices, knowing how the practices interact productively in argumentation, and recognising how each plays out in middle school language. Mathematics Studio was a productive forum for teachers to make progress on these challenges.

Both the math coach and associate principal played a key role in shaping teachers’ learning experiences. This was accomplished by making the group’s evolving understandings of CGJ explicit, challenging math learning goals to move lessons away from procedures, and requesting a rationale for pedagogical suggestions to encourage teachers to reflect on instruction through a CGJ lens.

As evidenced in the detailed Math Studio agendas, considerable effort was spent planning and developing protocols to outline how teachers’ ideas would be elicited, processed and recorded. The coach was a strong facilitator of these protocols and could move comfortably between facilitator and participant roles. The associate principal maintained an important participatory role by reinforcing the goals of Math Studio and articulating how those fit within broader school initiatives. Both pressed on teachers thinking, and consistently requested that teachers defend pedagogical decisions based on the potential to elicit CGJ. These moves made teachers’ tacit knowledge and beliefs open for debate and compelled the group to question standard instructional practices. Moreover, I contend that making this reasoning explicit increases the likelihood that teachers will be able to employ new strategies, such as asking students to judge the validity of a math argument rather than relying on the teacher’s evaluation, appropriately in the future.

Comparing Mathematics Studio to Lesson Study

The core components of lesson study, namely the collaborative planning, live observation and analysis of a classroom lesson are inherent in the Mathematics Studio model. However, there are some subtle distinctions between lesson study and the Mathematics Studio work described in this paper. Articulating these differences can contribute to our growing understanding of how the lesson study process can be successfully adapted to address localised needs (Fernandez, 2002; Perry & Lewis, 2009).

First, the focus of Math Studio, to promote CGJ, was pre-determined by the researcher in consultation with the math coach and building administration, rather than by the teacher group. Second, because the emphasis was on mathematical practices, the mathematics content

of lessons was often tangential. This is in contrast to Japanese lesson study in which the cycle typically begins with teachers articulating a goal for student learning followed by intense curricular study, or *kyozai kenkyu* (Lewis, Perry & Hurd, 2009; Yoshida, 2008).

Third, whereas a lesson study cycle may be sustained over the course of several weeks (Fernandez, 2002), each Math Studio cycle was condensed into two 3-hour sessions. This structure further precluded deep interrogation of the curricular resources or long-term lesson refinements. However, these adaptations supported the overall goal of increasing teachers' understanding of CGJ and ability to promote these practices in the classroom. Because we designed and observed multiple lessons across varied content, teachers saw how these mathematical practices permeate the curriculum. Moreover, this design afforded recurring feedback loops so that teachers could refine their understandings of CGJ and begin to generalise about pedagogical strategies that would elicit CGJ.

Finally, the purpose of the lesson planning process was not to devise a lesson that would be modelled or used by others. Instead, the intent was to increase the collective knowledge of the group with regard to what CGJ entailed and how to motivate or nurture these practices in the classroom. Granted, lesson study advocates would also argue that the goal of lesson study is not to devise the perfect lesson, but rather to improve teaching (Hiebert & Stigler, 2000; Yoshida, 2008). However, there is still an expectation that public lessons are continually refined and eventually shared with a broader audience. While the teachers in this study were encouraged to try out the Studio lessons and report out during the next cycle, there was no intentional plan to formalise what was learned or present to others outside the group. Granted, the highly contextualised nature of the Math Studio work makes public sharing seem somewhat unnecessary. However, this lack of documentation signals missed opportunities to solidify teachers' learning across the year and raises questions about sustainability.

Limitations

As with any case study, there are limits to the generalisability of the findings presented here. First, it is beyond the scope of this study to make rigorous claims about the extent to which teachers changed classroom practices. Data on participating teachers' instruction was limited to self-report, field notes compiled during brief (15-20 minutes on average) classroom observations between Studio cycles, and examples provided by the math coach and associate principal who frequented the classrooms more consistently. However, multiple data sources, including Studio surveys, teacher reflections and artefacts such as the CGJ matrix, provide consistent evidence that teachers increased their knowledge of CGJ and awareness of promoting these practices in the classroom.

Secondly, the combination of knowledge and productive dispositions towards mathematics and teacher learning modelled by the associate principal and math coach cannot be discounted. The associate principal, Sarah, was admittedly, "not a math person." However, she had a clear vision of the type of instruction she wished to promote and had taken it upon herself to increase her knowledge by attending mathematics professional development and building a strong relationship with the mathematics coach. Pam not only had a substantial background in mathematics, but was also well versed in the coaching literature. She emphasised inquiry, collaboration, teacher as decision-maker and continual adult learning; characteristics Saphier and West (2009) deem necessary to positively impact teaching and learning. The unique characteristics of Sarah and Pam may raise questions about how this work might be replicated. However, by detailing the contributions of these school-based leaders, my hope is that this expertise is not seen as residing in the individuals of this case, but rather as a guide to roles that need to be taken up by the group.

In terms of research, this study raises a number of questions to investigate further. For example, is there a minimum number of cycles or a trajectory of experiences that teachers need to substantially impact their own practice? Once teachers have embraced the principles and format of Mathematics Studio, how does the role of a math coach or principal diminish or shift? And finally, what knowledge, skills and experiences are necessary for teacher leaders, administrators and others to take up this work?

Conclusion

Lewis, Perry, Friedkin and Roth (2012) argue that one reason teachers have difficulty changing classroom practice is because they lack high quality instructional resources and practice-based opportunities to develop shared knowledge and commitments. This study documents ways in which Mathematics Studio provided a team of middle school teachers with such supports. The integration of outside readings with collaborative lesson planning, observations and analysis of student discourse raised teachers' awareness of intentionally planning for CGJ and supported a common vision of what these mathematical practices might look like in a classroom. By highlighting participant roles, these findings provide insight into how teacher learning about CGJ instructional practices can be fostered by school leaders. Such work is imperative if we are to make progress towards engaging all students in authentic mathematical conjecturing, generalising and justifying.

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