

MAINTAINING THE MATHEMATICAL FOCUS OF WHOLE-CLASS DISCUSSIONS: DILEMMAS AND INSTRUCTIONAL DECISIONS

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As teachers shift their practice from traditional student-to-teacher interaction patterns to collaborative discussions around rich mathematics, they often encounter instructional challenges. One such challenge is deciding when to pursue interesting and productive ideas that run contrary to the particular mathematical goal of the lesson. In this research summary, I report results from a study involving one experienced teacher's instructional decisions and how she maintained attention to the lesson-specific content goal given possible alternative pathways.

Keywords: Classroom Discourse, Instructional Activities and Practices

Over the past several decades professional mathematics teaching organizations and mathematics education research has described the potential benefits of classroom discourse for supporting students in meaningfully understanding mathematics. This attention to student thinking and discussions have important implications for students' opportunities to access mathematical ideas and their views of what it means to be mathematically competent. As such, many sets of instructional techniques have developed to support teachers in facilitating rich mathematics discussions. These foci have varied from identifying cognitively demanding tasks (e.g. Stein & Smith, 1998), developing instructional practice (Smith & Stein, 2011), the use of various talk moves (Chapin, O'Connor & Anderson, 2009), and attention to equity (Khisty & Chval, 2012), to name a few. Many of these approaches share the common goal of creating opportunities for equitable access to meaningful mathematics.

At the same time, it has been well documented that many U.S. teachers still use direct instruction approaches by asking simple-fact based questions, receiving responses from students, and then providing feedback or evaluation (Cazden, 1988; Chapin, O'Connor & Anderson, 2009; Franke, Kazemi & Battey, 2007; Sahin & Kulm, 2008; Stigler & Hiebert, 1999). Perhaps one reason for this more traditional approach is that by limiting student talk, there is a clearer pathway forward in a lesson, providing less opportunity for a student question or insight to potentially deviate the trajectory of the lesson. Additionally, researchers have contended that facilitating classroom discussions where students are collaboratively engaged around rigorous mathematical content is quite challenging (e.g. Franke, Kazemi, & Battey, 2007; Lampert & Cobb, 2003; Lampert et al., 2013; Tyminski, Zambak, Drake & Land, 2013), especially when teachers have multiple goals they are attempting to satisfy (Sleep, 2012). Lampert and colleagues (2010) for example, argued that, "Maintaining a coherent mathematical learning agenda while encouraging student talk about mathematics is perhaps the most challenging aspect of ambitious teaching" (p. 131).

As such, facilitating discussions has the potential to create instructional dilemmas as teachers grapple with advancing the mathematical agenda of the lesson while balancing possibly competing instructional goals such as developing student engagement in mathematical practice, supporting long-term content goals (e.g. understanding linear relationships), and establishing a mathematical community within the classroom. Because facilitating discussions around rich

mathematical tasks requires a different set of skills, this raises the question, as to what barriers still persist in supporting teachers with facilitating these types of mathematics discussions and how one might go about mitigating these barriers.

One approach to studying this phenomenon would be to study teachers who typically use direct instruction but are open to the idea of facilitating discussions. This could provide insight as to the reasons why these teachers have not yet taken the first step towards a shift in their instruction. In the study described in this article, I take a different approach, focusing on teachers who are well beyond their first step in facilitating discussions. The middle school teacher described in this research summary is an experienced teacher (having taught at other grade levels), is philosophically aligned with the value of student talk, and she used the *Connected Mathematics* (Lappan et al., 2005; Lappan et al., 2014) middle grades curriculum – a textbook series designed to engage students in thinking through student discourse.

The results described in this summary attend to instructional decisions during whole-class discussions where the decisions took place at a potential crossroads in the lesson, where the teacher could have chosen to pursue alternate goal-types but was able to maintain her attention to the lesson-specific content goals through her instructional choices. The results described in this research summary focus on this teacher's decisions when she was presented with situations which could have deviated her instruction from lesson-specific content goals as she instead chose to attend to broad content goals, mathematical practice goals, or goals for establishing a mathematical community.

Purpose

Broadly, the purpose of this study was to better understand the potential barriers and inherent complexity in teachers' decision-making while facilitating classroom discussions. More specifically, the results given in this proposal describe pivotal moments in lessons, where teachers chose to maintain the lesson-specific content goal instead of deviating and focusing on a different type of instructional goal. The hope is that by better understanding how experienced teachers navigated the complexity of attending to the lesson-specific content goal(s), that this might better inform the field as to the potential deviations that teachers are likely to encounter while facilitating whole-class discussions. In some ways, these potential deviations might be viewed negatively as they move the focus away from the mathematical goal within the lesson; however, the reason these posed actual dilemmas for teachers is because they are important aspects of the long-term development of students as thinkers, reasoners, and doers of mathematics. Thus, it is through understanding how experienced teachers supported these other goals while still maintaining focus on the lesson-specific content goal, that we might better understand how to support novice teaching practice in facilitating discussions.

Theoretical Framework

The guiding theoretical perspective for this study was *situated cognition* (Greeno, 1989). The situated perspective places strong emphasis on the interaction between individuals and their environment. As Greeno (1989) stated, "Cognition, including thinking, knowing, and learning, can be considered as a relation involving an agent in a situation, rather than as an activity in an individual's mind" (p. 134). In this study, teacher's activity and decisions were understood in the context of their activity in their classrooms. I attended to teachers' interactions with their students over multiple lessons, observing behavior, hypothesizing about their decisions, and later interviewing them about possible decisions. This approach differs from simply interviewing

teachers about a simulated classroom or asking teachers to state what decisions they would make in a theoretical situation in their own classroom. The approach I used in this study aimed to provide a more authentic view of teachers' decision-making as a cognitive activity necessarily embedded in their environment.

Additionally, I investigated the nature of teachers' decisions, by using the theoretical construct of *professional obligations* (Herbst & Chazan, 2011) which states that teachers' decisions are a function of their obligation to (a) students as individuals, (b) in developing students interpersonally in the classroom, (c) in representing the school as an institution and (d) in representing the discipline of mathematics. This construct helped in better understanding the nature of instructional decisions when teachers encountered instructional dilemmas, defined as a conflict between one of four instructional goal types (lesson-specific content goals, broad content goals, mathematical practice goals, and goals for establishing a mathematical community). A teacher's professional obligations differ from their goals in that goal(s) are about accomplishing a task (e.g., supporting students in effectively communicating their thinking with the class), whereas the teacher's professional obligation(s) describe who or what they feel obliged to satisfy (e.g., the individual student, the school, etc.) during the discussion.

Methods

To better understand the nature of teachers' instructional goals and identify the moments where potential conflicts occurred I analyzed data from a variety of sources, including an initial interview with each of the three grade-seven teachers, a baseline lesson observation, lesson plans for three rounds of at least three consecutive lessons per round, post-lesson interviews, and video-stimulated interviews at the conclusion of each round. To identify points at which goals may have been potentially in conflict with one another, I first conducted a preliminary analysis to identify teachers' instructional goals and determined moments when any consistent patterns of instruction deviated from what was observed. Across the three teachers in the study, I identified goals in all sources of the data. The instructional goal is defined as what the teacher hopes to accomplish as a result of enacting a whole-class discussion. Most goal statements occurred during interviews, where teachers were asked directly about their goals such as, "...were there like a few big things you really wanted to come out of [this lesson]?" or "Were there other goals, or other things you wanted..."

During the initial round of data analysis, I identified goal statements where the teacher either explicitly stated their goal or described their intended learning outcome. For example, during Ms. Mitchell's post-lesson interview (Round 2, Observation 2) she stated "...my goal as I told you was for them to get a better sense of what a sample space is." This is an example when the teacher explicitly identified her instructional goal. As a second example, Mr. Sandberg stated his goal in the form of an expected learning outcome, "...they need to be able to understand y-intercept, what that looks like on a table, graph, and equation" (post-lesson interview, Round 1, Observation 2).

After identifying goal statements, I coded goals as to whether the goal was primarily focused on (a) lesson-specific mathematical content, (b) mathematical content that was beyond the scope of an individual lesson: (long-term or unit-level), (c) mathematical practice goals (including goals attending to communication and dispositions) or (d) goals focused on being a member of a mathematical community. All teachers had cases of all four types of goals.

This first category, *lesson-specific content goals*, focused on mathematical content to be learned in a particular lesson. The second category *broad content goals*, included unit-level

content goals such as understanding linear relationships, a topic taught over several lessons. This category also included long-term content goals dealt with goals that went beyond the unit, such as learning goals that connected to the high school curriculum. This type of goal is strongly related to Ball's (1993) notion of teaching towards the "mathematical horizon".

Mathematical practice goals are consistent with NCTM's (2000) Process Standards and the Common Core's Standards of Mathematical Practice (NGA-CCSSO, 2010). This goal type is related to the first two goal types in that they focus on mathematics, but they differ in that they focus not on content, but on types of mathematical practices, such as justification and critiquing other students' arguments. The fourth category, *goals for establishing a mathematical community* included statements about developing a classroom that would support shared exploration and expectations of having students to learn in a social environment.

It is worth noting, that I do not make the claim that teachers are always aware of their goals or, if they are aware of them, that they are able to articulate them clearly. In many cases, teachers may have instructional goals that are somewhat unknown to them (e.g., a novice teacher knowing what they should be teaching a particular activity without a clear reason why they ought to be doing it). A teacher may be acting consistent with a goal, without knowing what their goal is. Some teachers, for example, might simply have a goal to make sure the class runs smoothly, and maintaining order is the most important aspect of their instruction with very few mathematical goals. I take as an assumption that the nature of teaching falls in the category of "goal-oriented activity," but I am cautious in not assuming that any discussion necessarily includes all of the four types of goals.

Results

In this section, I describe the results specifically pertaining to instances where Ms. Mitchell's goals actively competed with lesson-specific content goals. To illustrate these instances, I describe one instance of each of the three types of conflict between lesson-specific content goals and the other three goal types. While all three teachers had at least one conflict between the lesson-specific content goal and each of the other three goal-types, I provide data only on Ms. Mitchell as she had the most collective instances of conflicts between lesson-specific content goals and the other three types.

As an overview of her instructional approach, Ms. Mitchell could be best characterized as a teacher who thinks deeply about her teaching practice, as she often reflected on her instructional decisions in relation to developing content, practice, and dispositional goals for her students. More than once, she described the importance of social inequities and her role in teaching mathematics as a way to provide opportunities for students to grow both personally and academically. As such, she often found herself in various instructional dilemmas where lesson-specific-content goals potentially conflicted with other goal types.

In one of Ms. Mitchell's lessons, a student was describing the y-intercept as the "starting point" on the graph, and during the post-lesson interview she described how uncomfortable that made her feel. "When I first saw that [the y-intercept] I was like, oh, I don't like that one bit, 'cuz in high school I had to fight that for three years. It's not a starting point." Ms. Mitchell allowed the student to talk about the y-intercept as a starting point as they were just beginning to understand this important point across representations. This exchange highlighted Ms. Mitchell's attention to long-term content goals (broad-content), specifically related to her experience as a high school teacher.

Ms. Mitchell also frequently mentioned Common Core standards (NGA-CCSSO, 2010) at the beginning of class – both content and practice – and her use of CCSSM practice standards was a theme throughout her instruction both in describing her decisions and in making these standards explicit to students during class time. In one interview she mentioned wanting students to engage in repeated reasoning and thinking about structure. Other practices she mentioned included having students provide evidence to support a claim and persevering when trying to solve a problem. These statements reflected mathematical practice goals.

Ms. Mitchell also talked about the importance of protecting students' think time, and what students can learn from each other in groups (establishing a mathematical community). She wanted her students to be able to interact with each other directly in the discussion and for her role to be minimized in the discussion. In Ms. Mitchell's classroom this involved one lesson where she explicitly changed the typical norms of interaction taking a less prominent role in the classroom discussion allowing for greater student-to-student interaction.

Case 1: Lesson-Specific Content Goals vs. Broad Mathematical Goals

In this first lesson, students were analyzing their previous day's work from a problem involving the fundraising graphs for a walkathon by three fictitious students (Figure 1). The problem in the textbook (*Moving Straight Ahead*, p. 8, *CMP2*) gave the fundraising as follows: Leanne \$10 regardless of distance; Gilberto \$2 per km; Alana \$5 plus \$0.50 per km. The lesson-specific content goal of this lesson was for students to make and recognize linear tables, equations and graphs from contexts.

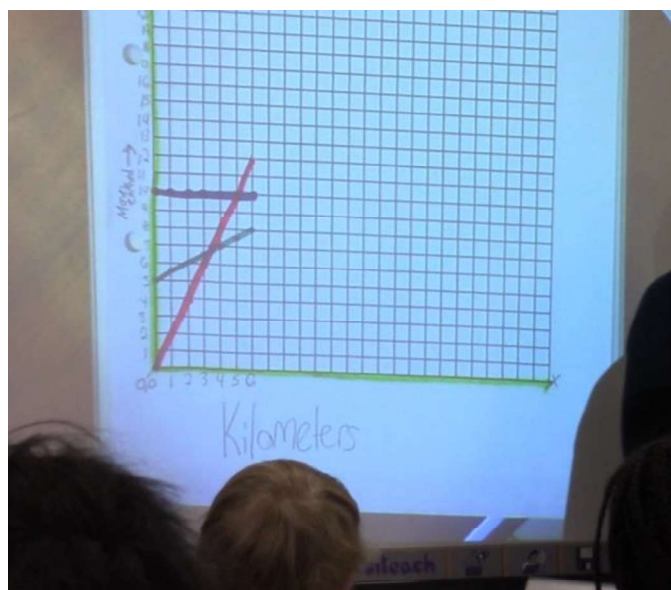


Figure 1: Ms. Mitchell Displays Walkathon Results

During the class discussion, Ms. Mitchell pressed a student to explain why the graphs were linear, after the student noticed the consistent shape in the three graphs. To contrast linear and non-linear graphs, Ms. Mitchell asked the student to draw a graph of a non-linear relationship on the board. Whereas the student may have simply drawn a non-linear function (e.g., a quadratic or exponential relationship), the student instead drew a graph that was not even a function (Figure 2). Given this strange graph, a different student asked, “Could a graph actually go backwards like that? Like could it go like this way and then eventually go back that way again?” This question

focused on an important relationship in understanding a mathematical function. At this point, Ms. Mitchell was faced with a decision of whether to (a) pursue this important mathematical insight, which was beyond the scope of the lesson, but was potentially productive in supporting students' thinking about functional relationships, or (b) refocus attention to the lesson-specific content goal.

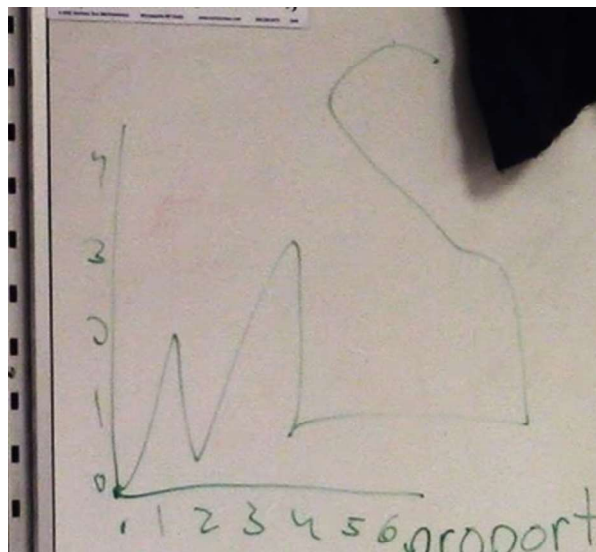


Figure 2: A Student Draws a Graph of a Non-linear Function

Ms. Mitchell could have moved on from this student's thought, minimizing the opportunity for disruption. Her decision was to write a note on the side of the board and to validate the importance of the student's insight. "I'm going to write that right here Charlie because that's outside what we want to talk about today. But that's an extremely important question; could a graph actually go backwards? And we need to talk about that."

During the video-stimulated interview after the first round of observations, Ms. Mitchell commented on this clip:

Charlie's question was so awesome as it usually is. That is an example where I don't want to shut down the conversation or, well I do want to shut down the conversation because the level of thinking it would take based on what they had had by one-two would be too great...I do want to push a little bit...Now that thing about going back on itself I still have that written down and we will come [back] to that.

Although Ms. Mitchell's decision was to move on from the student's question, she did this only after valorizing the student's question. Ms. Mitchell could have chosen to pursue the student's idea directly, essentially moving away from her lesson goals for the discussion. Instead, she refocused the discussion in line with the lesson goals. By supporting the student's thinking and publicly acknowledging its importance, she continued to make explicit the importance of thinking and questioning in her classroom.

Case 2: Lesson-Specific Content Goals vs. Goals for Establishing a Mathematical Community

This second case occurred as students explored a problem on compound probability. In this lesson, students rolled two six-sided dice and found the product of the two values. They did

several trials. One student won if the product was even; the other student won if the product was odd. This game would not be considered a “fair game”, because the likelihood of getting an even product is greater than an odd product. The lesson-specific content goal for this lesson was to develop an initial understanding of whether a game is fair or not.

During the discussion, Ms. Mitchell asked the class if they thought the game was fair. One student, Jonathon, said the game was fair because there are 3 odd outcomes and 3 even outcomes per die. This is incorrect since the outcome is based on the products of the two values, not just the individual values appearing. Ms. Mitchell was faced with how to appropriately respond to the student given that this particular student often has difficulties in the class. On the one hand, Ms. Mitchell needed to advance lesson-specific content goals, helping students see the inherent unfairness to this game. On the other hand, if her response was to be consistent with trying to establish a mathematical community, she would need to be careful about how she responded to a student who typically has difficulties in the class.

One choice would have been to press students to see why the claim was incorrect. She also could have simply stated that Jonathon was incorrect, or called on a different student in the class to explain why he was incorrect. Instead, Ms. Mitchell supported Jonathon’s thinking, and acknowledged how he was reasonable (in some way) in thinking that the game could be fair. The decision Ms. Mitchell made when faced with an incorrect student solution was to attend to the lesson-specific content goals and goals for establishing a mathematical community. During the discussion she had a student (Josh) sitting close to Jonathon talk to him about the odd number times odd number case, and later asked for other students’ thinking about the problem. She described this scenario during the post-lesson interview, describing her thought process during this moment in the lesson.

Jonathon, you know, he flat out fails; he doesn't have a whole lot of perseverance intellectually. He thinks of things sometimes really in strange ways, but I was happy that he was participating, and is in fact that half the numbers are even and half the numbers are odd. So I wanted to, you know, honor that he said, and then, but also try to build on that and then I felt like Josh kind of jumped ahead and wasn't real clear about odd times odd. So rather than just go there, and have him or someone explain how he knew that, I felt like the whole group would kind of get lost, so I just stated it very simply after he said it so people could think about it. Seemed to me there was still a question about the odd times odd. And they were all just kind of letting it go... (Ms. Mitchell, video-stimulated interview, May 11, 2015)

Case 3: Lesson-Specific Content Goals vs. Mathematical Practice Goals

A common dilemma for Ms. Mitchell was the need to meaningfully engage students in mathematical practice while also attending to lesson-specific content goals. This dilemma occurred across several lessons. Primarily, Ms. Mitchell worried about whether her students were developing their abilities in independent problem solving. During one lesson, students were learning how variability affects the mean and median of a data set, in relation to large clusters (the lesson-specific content goal). Throughout the lesson, there were several opportunities for Ms. Mitchell to focus on mathematical practice goals, but she continued to focus on the lesson-specific content goals due to the pressure of a standardized department final at the end of the month. She felt frustrated about having to choose one over the other given the content coverage expectations.

Well there's a dilemma, because on the one hand I would love to have all the ideas for them to have access to; on the other hand there's the bigger idea of the mathematical practices, like what do I do when I don't know what to do?...And, so I've got this treadmill keeping us moving toward the end at the same time as I'm trying to incorporate as much of the thinking stuff that I can. (Ms. Mitchell, post-lesson interview, May 19, 2015)

She later referred directly to how this dilemma impacted that day's lesson and the previous day's lesson, wondering if students saw the mathematical purpose of what they were doing. She also worried about the long-term impact of her instructional decisions to focus on the lesson-specific content goals in order to satisfy a common department final at the sacrifice of developing students' self-efficacy in problem solving. In contrast to the two previous cases, where Ms. Mitchell was able to navigate the potential conflict between lesson-specific content goals and other instructional goals, this case highlights that not all potential conflicts end in a productive resolution even for an experienced teacher. The institutional constraints of expectations for content coverage, standardized assessments, and time constraints was sufficiently pervasive to cause challenges for Ms. Mitchell.

Conclusion

During whole-class discussions, teachers are faced with decisions related to the types of questions they ask students, which student ideas they choose to highlight, and how deeply to pursue a student response. As teachers facilitate discussions, the dilemmas they encounter and the decisions they make send messages (explicitly or implicitly) to students about what is valued as thinkers and doers of mathematics. These messages are important because as Jansen (2006, 2008) pointed out, students participate in discussions for various reasons, yet student participation is a necessary component of teaching using whole-class discussions.

As was seen in the three cases provided in this research summary, Ms. Mitchell's experience in facilitating discussions and her orientation to balancing several competing instructional goals allowed her to make decisions that maintained the focus on the lesson-specific content goals. Much like other approaches to supporting teachers in facilitating discussions which include sets of instructional practices (e.g. Smith & Stein, 2011) and talk moves (e.g., Chapin, O'Connor, & Anderson, 2009) this study highlights that there are particular sets of technical skills needed to make in-the-moment decisions which likely impacted the immediate and long-term experiences and opportunities for students to learn mathematics via classroom discussions. In the findings reported in this summary, Ms. Mitchell showed particular techniques for how to navigate potential situations as they appeared in the course of an open dialogue with her students. These particular techniques might not all be obvious or clear to teachers who are shifting their practice from more traditional interaction approaches to a more open discussion format.

This study has the potential to further complexify existing sets of instructional techniques as well as to develop its own set of categories of situations a teacher may potentially encounter while facilitating discussions, such as, responding to an incorrect answer, responding to a question that moves beyond the scope of the lesson, or responding to an answer that other students do not understand.

As one such example, one of Smith and Stein's (2011) *Five Practices* is anticipating student thinking. In the findings in this study, one might consider particular likely scenarios, such as the Case 1 situation, where a student raised an interesting mathematical question that is relevant to the lesson but would likely move the focus of the discussion elsewhere. The teacher could

anticipate this particular situation and consider how they would generally approach a similar scenario. The teacher could consider under what conditions it would be appropriate to deviate from the lesson goal, or alternatively, how one refocuses the class on the more immediate lesson goal while still attending to the productiveness of the question and the student's engagement. By identifying and naming these types of situations, teachers that are shifting their practice towards increasing the quality of student talk in their classrooms may be better positioned to make in-the-moment decisions that allow them to maintain their lesson-specific content goal while also addressing other valuable instructional goals.

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