EXTENDING APPROPRIATENESS: FURTHER EXPLORATION OF TEACHERS' KNOWLEDGE RESOURCES FOR PROPORTIONAL REASONING

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In this study we extend our prior exploration focused on the extent to which middle school teachers appropriately identified proportional situations and whether there were relationships between attributes of the teachers and their ability to identify proportional situations. For this study, we analyzed both a larger dataset (n=32) and two dynamic scenarios in which participants were asked to consider aspects of the relationship shown in the diagrams. We found teachers who were correctly able to discern that a situation was not proportional were more likely to use important knowledge resources to evaluate the tasks.

Keywords: Teacher Knowledge, Rational Numbers, Mathematical Knowledge for Teaching

Purpose and Background

Proportional reasoning is an important mathematical concept in middle school mathematics. Despite its prominence in both the mathematics (National Governors Association & Council of Chief State School Officers, 2010) and science standards (NGSS Lead States, 2013), proportional reasoning has not enjoyed a rich history of research relative to its importance (e.g., Lamon, 2007). Research available on teachers' understanding is sparse, but indicates that, like students, teachers struggle with proportions (e.g., Akar, 2010; Harel & Behr, 1995; Izsák & Jacobson, 2017; Orrill, Izsák, Cohen, Templin, & Lobato, 2010; Post, Harel, Behr, & Lesh, 1988; Riley, 2010).

One necessary element of a robust understanding of proportions for teachers is the ability to distinguish those situations that are proportional from those that are not. Orrill et al. (2010) observed that the middle school teachers in their studies had trouble identifying situations as appropriate or inappropriate for using proportional reasoning. For example, when teachers were given a problem with three values and asked to find a missing fourth value, teachers tended to treat those situations as directly proportional even if the actual relationship was inversely proportional or linear. Teachers also struggled to apply proportional reasoning in a qualitative task (e.g., one that does not rely on manipulating numbers) that asked them to compare one pile of blocks to another pile, similar to those tasks used by Harel, Behr, Post, and Lesh (1992), instead they relied on additive reasoning.

Such findings led us to wonder how pervasive these issues were, what kinds of situations might confuse teachers, and what knowledge teachers rely on to determine whether a situation is proportional. In this paper, we extend our earlier findings (Nagar, Weiland, Brown, Orrill, & Burke, 2016) related to this topic by looking at data from more teachers and by expanding our task set to include a dynamic task that appropriately modeled a proportional relationship with a "thermometer" representation (see Figures 1 & 2). Specifically, we consider which knowledge resources were most frequent and what trends emerged among teachers who were able to differentiate proportional from

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non-proportional relationships versus those who struggled to do this. This work is at the crossroads because it brings together theory and practice in a way that is expressly aimed at impacting practice. By understanding how teachers think about proportional situations, we are better able to create teacher professional development experiences that meet the teachers where they are, thus maximizing the potential for impacting students' experiences with mathematics.

Theoretical Framework

We work from the knowledge in pieces perspective (diSessa 1988, 2006), which asserts that individuals develop understandings of various grain sizes that are used as knowledge resources in a given situation. These resources are connected, over time, through learning opportunities that lead to the refinement of the resources and the development of rich connections. More rich connections between knowledge resources allow them to be available in more situations. This is parallel to the research on expertise that has shown experts have both more knowledge and a different organization of knowledge than novices in their domain (e.g., Bédard & Chi, 1992). It is also aligned with Ma's (1999) interpretation the need for teachers to have profound understandings of fundamental mathematics. By having a robust set of knowledge resources that are coherently connected, we posit teachers will be more able to access their understandings to apply them to a wider range of mathematics and teaching situations than others whose knowledge resources are less coherently connected. We refer to this richly connected collection of knowledge resources as being *coherent* and assert that more coherent teachers will be better able to support student learning (e.g., Thompson, Carlson, & Silverman, 2007). This approach differs from much research on teacher knowledge in that we are not trying to identify deficiencies in teachers' understanding of mathematics, rather, we are trying to understand how teachers understand the mathematics they teach and how different knowledge resources are drawn upon for solving problems and teaching.

Methods

This study is part of a larger project investigating teachers' knowledge of proportional reasoning for teaching. The participants included a convenience sample of 32 in-service, grade 5-8 mathematics teachers, whose teaching experiences ranged from one to 26 years. The participants were from four states. They taught at a variety of schools (public, private, and charter). Twenty-four of the teachers identified as female and eight identified as male. Six of the teachers identified as a race other than white.

The data analyzed for this study were collected through a task-based clincial interview that was videotaped using two cameras trained on the participant's hands to ensure we captured anything the participant wrote or pointed to in the interview. Each interview lasted about 90 minutes. Additional data were collected in the form of a written assessment of proportional reasoning that included the LMT Proportional reasoning instrument (e.g., Hill, 2008) augmented by additional questions focused specifically on whether participants could discern proportional situations from non-proportional situations.

The qualitative analysis of the participant's clinical interview responses was carried out by coding the participants' utterances using a coding scheme that was developed using emergent coding focused on the knowledge resources participants used to reason about a variety of situations. This coding scheme, which included of 23 codes, relied on codes from the literature (e.g., Lobato & Ellis, 2010) as well as from open coding (Corbin & Strauss, 2007). This approach of relying on both literature and emergent codes is consistent with certain grounded theory approaches (e.g., Charmaz, 2014). To create the coding scheme, we coded several interviews, with 2-5 members of the team coding each interview until we were certain that the coding scheme included all the relevant resources we were observing. The full coding scheme included knowledge resources related to

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reasoning about ratios and proportions, the relationship between fractions and ratios, the relationship between similarity and proportions, the use of representations to reason about proportions, and a few pedagogically-related code, such as one to capture those instances in which a teacher indicated she would ask the student for additional information. For the purposes of this study, we present only those codes that appeared across both studies (shown in Table 1) specifically relevant to proportions (e.g., excluding those for representations and pedagogy). Our coding relied on a binary approach in which each utterance was coded as a 1 or a 0 based on whether a particular knowledge resource was observed. Once the coding scheme was stable, each interview was coded by at least two researchers and 100% agreement was reached on all coding.

Table 1: Codes of Knowledge Resources Used in Thermometers Task

Code	Description
Comparison of Quantities	States that ratio as a comparison of two quantities.
Multiplicative Comparison	Participant sees that there is a way of describing the relationship of the quantities in the ratio that is multiplicative
Covariance	Recognizes that as one quantity varies in rational number the other quantity must covary to maintain a constant relationship.
Unit Rate	Uses the relationship between the two quantities to develop sharing-like relationships such as amount-per-one or amount-per-x.
Equivalence	Describes proportion as a relationship of equality between ratios or fractions.
Constant Ratio	Recognizing the invariant multiplicative relationship between two quantities.
Scaling Up/Down	Uses multiplication to scale both quantities to get from one ratio in an equivalence class to another.
Horizon knowledge	Demonstrates knowledge that extends into mathematics beyond proportions
Rule	Shares a verbal or written rule (e.g., Red = Blue - 2) stated in a way that conveys a generalizable relationship.

For this analysis, we revisited the Thermometers task from our earlier study (Nagar et al, 2016). The thermometers task relied on a dynamic sketch presented to participants with two thermometers, one red and one blue, whose lengths could be varied by dragging a point on a number line (as shown in Figure 1 and Figure 2). Two scenarios were shown to participants (one at a time) and with each scenario participants were asked: (a) whether there was a relationship between the thermometers; (b) whether the relationship was proportional; (c) whether they could provide a rule and a story problem or real-world situation for that relationship; and (d) whether they see a scale factor involved in the situation. For the original study, we analyzed 13 participants' responses to the first scenario in which the thermometers were designed to maintain a constant difference of two units in length of the lines as the point on the slider is dragged from left to right (Figure 1). This situation represents a non-proportional linear relationship between the two thermometers. Our earlier findings showed that five of the 13 teachers initially misidentified the situation as proportional. In that analysis, we also found that two teachers (Group 3) remained convinced that the situation shown in Figure 1 was proportional, whereas the other three teachers (Group 2) started out thinking it was proportional, but then changed their mind. The eight teachers in Group 1 started out, and remained, convinced that the situation was not proportional. We then analyzed which knowledge resources the teachers relied on to determine that the situation was not proportional. Our analysis showed that teachers in Groups 1 and 2 used *Rules*, *Scaling Up/Down*, and *Equivalence* to appropriately identify this Thermometers task as non-proportional. We also found that across all three groups, teachers used language that sounded very additive rather than relying on multiplicative reasoning language.

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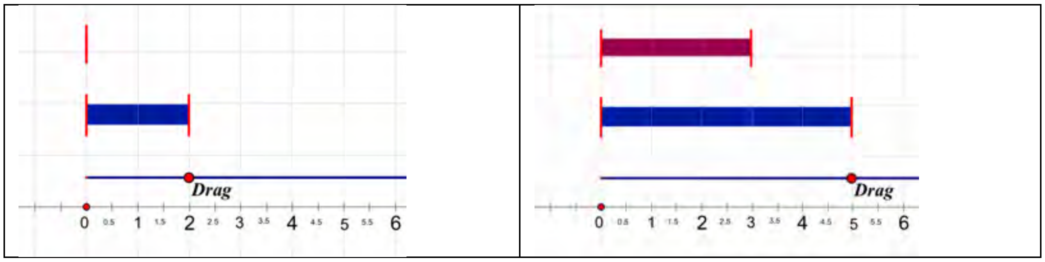

Figure 1. Screenshot of thermometers scenario 1 task.

In the study reported here, we extend the earlier work in two ways. First, we now have our entire dataset analyzed for the Thermometers Scenario 1 task (Figure 1), therefore, we consider 32 teachers' responses to that item (this includes the 13 teachers in the original study plus 19 additional teachers). Second, we analyzed Scenario 2 in the Thermometers task, a situation in which the dynamic environment models a proportional relationship (see Figure 2).

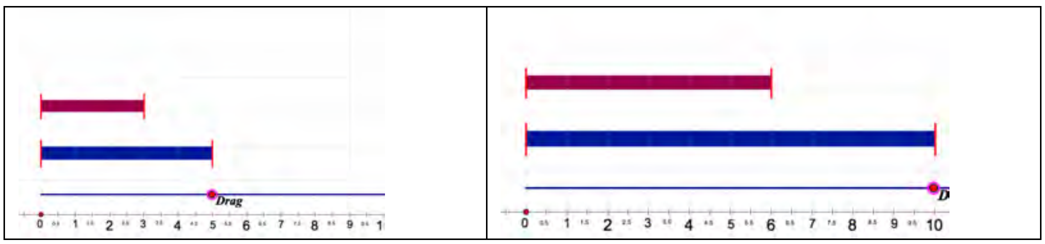

Figure 2. Screenshot of thermometers scenario 2 task.

Results

Our driving research question for this study was: what knowledge resources do teachers seem to rely upon in determining whether a situation is proportional or not proportional? We will first consider this question for Scenario 1 (the non-proportional situation), then for Scenario 2 (the proportional situation). In both scenarios, we focus on trends in the groups. All names reported in this section are pseudonyms.

Scenario 1: Linear Relationship

We began by separating the participants into groups the same way we had in the earlier study. The analysis of 32 teachers in the non-proportional Scenario 1 task showed that 19 teachers (59%) correctly identified the situation as non-proportional (Group 1). Seven participants (22%) first identified the situation as proportional but changed their mind during the interview to identify the situation as non-proportional (Group 2). And, six participants (19%) identified the situation as proportional (Group 3).

As in our earlier study the subset of codes shown in Table 1 were used in making sense of the situation. The most notable trend in the dataset was that the Group 3 teachers relied very little on these knowledge resources to make their determination. In fact, only three members of Group 3 (50%) used any of these resources. Peter used both *Unit Rate* and *Equivalence* while David used only *Equivalence* and Bridgette used *Horizon Knowledge*. In contrast, in Group 1, only four teachers (21%) did not use the knowledge resources included in this analysis. And, across the teachers there was much more variation with at least one person using each of the listed knowledge resources at least one time. For Group 2, two of the teachers (29%) did not use any of the resources.

Table 2: Number of Occurrences of Each Code by Group for Each Scenario

	Scenario 1				Scenario 2			
Code	Group 1 (*n*=19)	Group 2 (*n*=7)	Group 3 (*n*=6)	*Total (n=32)*	Group 1 (*n*=19)	Group 2 (*n*=7)	Group 3 (*n*=6)	Total (*n*=32)
Comparison of Quantities	1	0	0	1	3	1	0	4
Multiplicative Comparison	0	4	0	4	13	2	4	19
Covariance	3	1	0	4	5	1	1	7
Unit Rate	2	0	1	3	5	3	0	8
Equivalence	5	2	2	9	3	2	1	6
Constant Ratio	4	2	0	6	11	1	3	15
Scaling Up/Down	6	2	0	8	12	4	1	17
Horizon knowledge	5	3	2	10	5	1	1	7
Rule	21	6	9	36	12	4	1	18

Consistent with our earlier study, *Scaling Up/Down, Equivalence,* and *Rule* were some of the most used knowledge resources on this task. We note that *Scaling Up/Scaling Down* was not used at all by Group 3. It was used somewhat consistently in Group 1 with five of 19 teachers (26%) using it a total of six times. In Group 2, only one teacher out of seven (14%) used *Scaling Up/Down* in her reasoning twice. *Equivalence* was used nine times across all the teachers for Scenario 1, making it the third most commonly used code. In Group 1, three (Diana, Greg, Larissa) of the teachers used *Equivalence* a total of five times. In Group 2, two teachers each used it one time, and in Group 3, two teachers used is one time each.

Interestingly, the two most commonly used codes for this larger dataset on Scenario 1 were *Rule* and *Horizon Knowledge*. An example of *Horizon Knowledge* in this context would be recognizing that Scenario 1 is not a proportional relationship because the *y*-intercepts for the blue and red bars differ, as Alan did:

> Oh, as an eighth grade math teacher you'd say they have the same slope but a different Y intercept. Yeah, I know it's probably not what you're thinking about but, yeah when you go all the way back here it's, this is always going to be two ahead. So that starts a zero, this starts two, but then they grow at the same rate, so it's always two ahead.

The commonality of the *Horizon Knowledge* code is interesting as it suggests that having more formal understandings of mathematical structures to be able to generalize might matter in Scenario 1. It also suggests that understanding related mathematical topics may support teachers in invoking knowledge resources to better understand a given situation.

The *Rule* code was the most frequently observed in this coding scheme for Scenario 1. In Group 1, 14 of the teachers (74%) were able to generate a rule describing the relationship. For example, Diana said, "Red plus 2 would be blue." In Group 2, this dropped to three of seven teachers (43%). However, Group 3 had four of six teachers (67%) able to generate a rule relate to the situation. This suggests that the teachers in Group 3 may not be connecting their knowledge resources for determining whether something is a proportion to their generalization of a mathematical situation. For example, Brianna (in Group 3) clearly stated that the relationship was, "Whatever red is plus 2 would equal blue" but, she maintained that the relationship was proportional. This suggests that explicitly understanding the mathematical structure of the problem may not be necessary to generate a rule about the relationship presented.

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Scenario 2: Proportional Relationship

In Scenario 2 (Figure 2), we asked the same questions of our participants about a similar dynamic representation that showed a proportional relationship. Many more of the teachers got this task correct. In fact, only eight teachers (25%) gave wrong responses and two of those changed their response to be correct during the course of the interview (one in Group 2 and one in Group 3). Of the teachers who answered incorrectly and did not change to a correct interpretation, three were in Group 1, one was in Group 2, and two were in Group 3. This is consistent with our earlier finding that teachers have an easier time recognizing situations that are proportional than those that are not proportional (Nagar et al., 2016).

While *Scaling Up/Down* and *Rule* continued to be important in Scenario 2, *Equivalence* became less important and two new codes became more important: *Multiplicative Comparison* and *Constant Ratio*. *Multiplicative Comparison* was used only when an utterance demonstrated the participant understood a relationship between the quantities of the ratio as multiplicative. For example, understanding the blue thermometer is 5/3 as long as the red thermometer. While only one person (Kanita in Group 2) used *Multiplicative Comparison* as a resource for Scenario 1, in Scenario 2, 12 participants (38%) used it one or more times. In Group 1, nine participants used this knowledge resource 13 times for Scenario 2. In Group 2, two participants used *Multiplicative Comparison* two times in Scenario 2. Only one member of Group 3 (Patricia) used *Multiplicative Comparison* four times.

Constant Ratio was coded when participants indicated there was a fixed relationship between the two numbers in a ratio. It was not as precise as *Multiplicative Comparison* in that participants needed only to note the relationship existed without specifying the nature of that relationship (i.e. that it is multiplicative). In Scenario 1, six participants (19%) used this knowledge resource whereas 15 participants (47%) used it in Scenario 2. In Group 1, this was used eleven times across nine participants (47%) in Scenario 2. For Group 2, it was used just one time, and in Group 3, it was used three times by one participant (Patricia). This trend in *Constant Ratio* and *Multiplicative Comparison* codes suggests that there is something different about the way many members of Group 1 use their knowledge resources than the members of Groups 2 and 3. We note that Patricia in Group 3 appears to be an outlier in terms of her use of knowledge resources.

Generating rules was harder for teachers in Scenario 2 than in Scenario 1, but was still an important code with 18 instances across all three groups. For Scenario 2, six Group 1 teachers (32%) generated 12 rules, four Group 2 teachers (67%) generated four rules, and one Group 3 teacher (Patricia) generated one rule (17%). An example of one teacher's rule was Ella's, "So if I say the red bar is 3/5 of the distance to the blue bar, so the blue… so the blue bar… let me see if five… I don't want to like change this up. Five equals… so that would be like B=5/3R." The relative struggle the participants experienced in identifying a rule is interesting given that teachers were more successful identifying the situation as being proportional and reinforces our assertion that teacher knowledge is shaped by the specific context.

Conclusions

Consistent with our earlier findings, this study showed teachers are better at determining whether a situation is proportional if it is actually proportional. In the current study, 13 of the 32 teachers started out believing Task 1 was a proportion and only seven changed their thinking to recognize that the situation was not proportional. In contrast, only eight teachers were unable to initially identify a proportional situation as such, with two of those eight figuring out the situation was proportional as they worked. This is consistent with research on students that also shows problems discerning proportional relationships from non-proportional ones (De Bock, Van Dooren, Janssens, & Verschaffel, 2002).

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When this finding is combined with the particularly sparse use of relevant knowledge resources by teachers in Group 3, one reasonable assertion would be that teachers need more opportunities to apply their understandings and make connections between those understandings. For example, we are confident that middle school teachers understand ratios must be equivalent for a proportion to exist. However, few teachers applied this understanding, which could have helped provide evidence that Scenario 1 was not a proportion.

We also note that there may be some need for additional development of knowledge resources. For example, *Multiplicative Comparison*, which is a critical understanding for reasoning about the relationships within a proportion, was seen in a relatively small number of utterances. In Group 1, almost 2/3 of the teachers used it, but in Groups 2 and 3 combined, the resource was used by only three teachers. This suggests that having the *Multiplicative Comparison* resource available may lead teachers to be more accurate in their ability to discern proportional relationships. It also suggests that several teachers are lacking, or failing to activate, a critical understanding of proportional reasoning.

We assert that the lack of presence of *Multiplicative Comparison* may also explain the limited presence of the *Rule* code in Scenario 2, while it was very prevalent in Scenario 1. It may be that in teacher preparation and professional development teacher educators are over-emphasizing linear situations rather than multiplicative ones. It is also possible that the lack of comfort with the multiplicative relationship between quantities, implied by the limited use of the *Multiplicative Comparison* code, prevents teachers from seeing applications of proportional relationships in the real world. Perhaps focusing more on problem generation and the multiplicative nature of the relationship between quantities in a proportion would strengthen teachers' abilities to recognize proportional situations.

Combined, the findings of this study intersect theory with research to inform practice. By looking at teachers' actual use of knowledge through the knowledge in pieces lens, we are able to suggest that professional development be sensitive to both knowledge resources development and the development of connections between and among those resources. Failure to address both of these approaches creates a situation in which teachers are unable to capitalize on the knowledge they have to support their students.

Acknowledgment

The work reported here was supported by the National Science Foundation under grant DRL-1054170. The opinions expressed here are those of the authors and may not reflect those of the NSF.

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